International Journal of Systems Science

ISSN: 0020-7721 (Print) 1464-5319 (Online) Journal homepage: https://www.tandfonline.com/loi/tsys20

Stability and ℓ_1-gain analysis for positive 2-D Markov jump systems

Yong Chen, Chang Zhao, James Lam, Yukang Cui & Ka-Wai Kwok

To cite this article: Yong Chen, Chang Zhao, James Lam, Yukang Cui & Ka-Wai Kwok (2019) Stability and ℓ_1-gain analysis for positive 2-D Markov jump systems, International Journal of Systems Science, 50:11, 2077-2087, DOI: 10.1080/00207721.2019.1645229

To link to this article: https://doi.org/10.1080/00207721.2019.1645229

Published online: 02 Aug 2019.

INTERNATIONAL JOURNAL OF SYSTEMS SCIENCE
2019, VOL. 50, NO. 11, 2077–2087
https://doi.org/10.1080/00207721.2019.1645229

Stability and ℓ_1-gain analysis for positive 2-D Markov jump systems

Yong Chen[a], Chang Zhao [a], James Lam [a], Yukang Cui[a,b] and Ka-Wai Kwok[a]

[a]Department of Mechanical Engineering, The University of Hong Kong, Hong Kong; [b]College of Mechatronics and Control Engineering, Shenzhen University, Shenzhen, People's Republic of China

ABSTRACT
This paper investigates the problems of stability and ℓ_1-gain analysis for positive 2-dimensional (2-D) Markov jump systems. The mathematical model of 2-D Markov jump systems is established based on the Roesser model. Necessary and sufficient condition for stability and sufficient condition for ℓ_1-gain computation are derived. Furthermore, the stability and ℓ_1-gain conditions are extended to Markov jump systems with partially known transition probabilities. The effectiveness of the obtained theoretical findings is verified through two numerical examples.

ARTICLE HISTORY
Received 31 May 2018
Accepted 14 July 2019

KEYWORDS
2-dimensional systems; ℓ_1-gain; Markov jump systems; positive systems; stability analysis

1. Introduction

The 2-D model, in which the system state depends on two independent variables, has been used in different fields to represent a wide range of practical systems. This type of systems can be found in image data processing and transmission, thermal processes, gas absorption, and water stream heating. Due to their theoretical and practical importance, 2-D systems have attracted much research attention in recent years. The stability problem of 2-D systems has been investigated in Anderson, Agathoklis, Jury, and Mansour (1986), Lu and Lee (1985), and Hinamoto (1997). Two popular models of 2-D systems introduced by Roesser (1975) and Fornasini and Marchesini (1976, 1978) have been considered in those papers. The H_2 and H_∞ control problems, which aim to study the system performance, have been considered in Du, Xie, and Zhang (2001) and Yang, Xie, and Zhang (2006) for 2-D systems.

Besides 2-D systems, the variables of many dynamic systems are always confined to the positive orthant in many real processes, such as the drug therapy scheduling problem in HIV infection (Hernandez-Vargas, Colaneri, & Middleton, 2014). Some research studies have been focusing on positive systems. For instance, in Zhang, Jia, Zhang, and Zuo (2018), a new model predictive control framework has been proposed for positive systems with polytopic uncertainty. The exponential stability, ℓ_1-gain analysis problem and ℓ_1-induced controller for positive Takagi–Sugeno (T-S) fuzzy systems have been investigated in Du, Qiao, Zhao, and Wang (2018) and Chen, Lam, and Meng (2017). In Zhu, Wang, and Zhang (2017), the stochastic finite-time ℓ_1-gain filtering problem for discrete-time positive Markov jump linear systems with time-delay has been analysed. The dominant pole assignment problem, the dominant eigenstructure assignment problem and the robust dominant pole assignment problem for linear time-invariant positive systems have been considered in Li and Lam (2016). Some novel techniques have been developed in the literature to study positive systems, for example, the linear co-positive Lyapunov function method. As the state of a positive system is nonnegative, a linear co-positive Lyapunov function has been used in Lian, Liu, and Zhuang (2015) to analyse the mean stability of the positive Markov jump systems with switching transition probabilities. In addition, although the ℓ_2-gain and H_∞ performance index are important for characterising system performance of general systems, the 1-norm of the system state and the ℓ_1-gain of the system can provide more useful descriptions for positive systems (Chen, Lam, Li, & Shu, 2013; Lian et al., 2015; Shen & Lam, 2016; Xiang, Lam, & Shen, 2017). The 1-norm of the system state has a good physical meaning for positive systems because it is the sum of the values of the components of the state, for example, the 1-norm represents the sum of the number of materials if a positive system is used to model the number of the materials in a chemical process. The monograph (Kaczorek, 2012) provides a brief introduction for positive 2-D systems. In this monograph, some mathematical models (including the general model, the Roesser model, and the Fornasini-Marchesini model) of positive 2-D systems have been introduced, the controllability, minimum

CONTACT Chang Zhao zhaochang@connect.hku.hk

energy control, and realisation problems have been investigated there. Moreover, the author of the monograph (Kaczorek, 2012) also considered the problem of stability for positive 2-D systems with delays in Kaczorek (2009). In Kaczorek (2007), different forms of Lyapunov functions have been developed to study the positive 2-D Roesser systems. The stability problem for Roesser model has been treated in Kurek (2002). The authors in Fornasini and Valcher (2005) have focused on the controllability and reachability problems for positive 2-D systems by using a graph-theoretic approach.

On the other hand, many practical systems in the real world have multi-mode features, such as the HIV viral mutation model and traffic congestion model in Wang and Zhao (2017). Their multi-mode dynamics can be captured by the so-called switched systems. Recently, many research results concerning 2-D switched systems have been presented in the literature. The authors in Xiang and Huang (2013) have investigated the problems of stability and stabilisation of 2-D switched Roesser systems under average dwell time switching signal. The same problems for the Fornasini–Marchesini model have been considered in Wu, Yang, Shi, and Su (2015). In addition, the asynchronous control problem of 2-D switched systems under mode-dependent average dwell time has been considered in Fei, Shi, Zhao, and Wu (2017). As a special class of switched systems, Markov jump systems are able to model practical systems subject to abrupt changes. The switching in a Markov jump system is governed by a Markov process in the continuous-time case and a Markov chain in the discrete-time case. The stabilisation and H_∞ control problems of 2-D Markov jump Roesser systems have been studied in Gao, Lam, Xu, and Wang (2004). Wu, Shi, Gao, and Wang (2008) have studied the H_∞ filter design problem for the same class of systems. In Liang, Pang, and Wang (2017), sufficient conditions are established for the filtering error system such that the system is stochastically asymptotically stable with ℓ_1-gain. To the best of the authors' knowledge, there are not many research results on positive 2-D Markov jump systems in the literature. This motivates us for the present study.

The main contributions of this paper are summarised as follows:

(1) A necessary and sufficient condition for stability analysis for positive 2-dimensional (2-D) Markov jump systems is proposed for the first time.
(2) Sufficient condition for ℓ_1-gain computation is derived.

The rest of this paper is organised as follows. The considered dynamic systems and problems are formulated in Section 2. The assumptions, definitions, and lemmas that are used to derive the main results are also given in that section. The main results on stability and ℓ_1-gain computation are presented in Section 3. Numerical examples are given in Section 4 to verify the effectiveness of the theoretical findings. Finally, the conclusion of this paper is given in Section 5.

Notations: The notations used throughout this paper are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{N}_0$ denotes the set of nonnegative integers. $\mathbb{E}(*)$ denotes the expectation operator. '$\otimes$' denotes the Kronecker product. The superscript 'T' represents matrix transpose. $x \succ 0$ ($x \succeq 0$) and $A \succ 0$ ($A \succeq 0$) mean that all elements of vector x and matrix A are positive (nonnegative). $\mathbf{1}$ represents the vector $[1, 1, \ldots, 1]^{\mathrm{T}}$. I_n stands for the identity matrix. The 1-norm $\|x_{i,j}\|_1$, where $x_{i,j} = [x_{i,j,1}, x_{i,j,2}, \ldots, x_{i,j,n}]^{\mathrm{T}} \in \mathbb{R}^n$, is defined as $\sum_{l=1}^{n} |x_{i,j,l}|$. A vector $\omega_{i,j}$ belongs to $l_1\{\mathbb{N}_0, \mathbb{N}_0\}$ means that $\|\omega_{i,j}\|_1 < \infty$. Vectors and matrices are assumed to have compatible dimensions for algebraic operations if their dimensions are not explicitly stated.

2. Problem formulation and preliminaries

The considered positive 2-D discrete-time systems in the Roesser model with Markov jump parameters can be described as follows:

$$\mathcal{S}: \begin{bmatrix} x^h_{i+1,j} \\ x^v_{i,j+1} \end{bmatrix} = A(r_{i,j}) \begin{bmatrix} x^h_{i,j} \\ x^v_{i,j} \end{bmatrix} + B(r_{i,j})\omega_{i,j},$$

$$y_{i,j} = C(r_{i,j}) \begin{bmatrix} x^h_{i,j} \\ x^v_{i,j} \end{bmatrix} + D(r_{i,j})\omega_{i,j}, \qquad (1)$$

where $x^h_{i,j} \in \mathbb{R}^{u_1}$, $x^v_{i,j} \in \mathbb{R}^{u_2}$ are the horizontal and vertical state vectors, respectively; $\omega_{i,j} \in \mathbb{R}^v$ is the disturbance vector which belongs to $l_1\{\mathbb{N}_0, \mathbb{N}_0\}$; $y_{i,j} \in \mathbb{R}^w$ is the measured output vector. $A(r_{i,j}) \succeq 0$, $B(r_{i,j}) \succeq 0$, $C(r_{i,j}) \succeq 0$, $D(r_{i,j}) \succeq 0$ are positive real-valued system matrices. They are determined by an homogenous Markov chain $r_{i,j}$, which takes values in a finite set $\mathcal{L} = \{1, \ldots, S\}$ with transition probabilities

$$\Pr\{r_{i,j} = n, i+j = k+1 \mid r_{i,j} = m, i+j = k\} = \pi_{mn}, \qquad (2)$$

where $\pi_{mn} \geq 0$ and $\sum_{n=1}^{S} \pi_{mn} = 1$. Denote the transition matrix by $\Pi = \{\pi_{mn}\}$.

Remark 2.1: In this paper, it is assumed that switching occurs only at each sampling point of i or j. In other words, the value of $r_{i,j}$ depends on $i+j$ (Benzaouia, Hmamed, Tadeo, & Hajjaji, 2011; Duan & Xiang, 2014).

Denote system matrices $A(r_{i,j})$, $B(r_{i,j})$, $C(r_{i,j})$, $D(r_{i,j})$ as A_m, B_m, C_m, D_m, respectively, if the considered system operates at the mth mode, namely, $r_{i,j} = m$. The state of the system is denoted as

$$x_{i,j} = \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \in \mathbb{R}^u. \tag{3}$$

The boundary condition is (X_0, R_0) with

$$X_0 = \begin{bmatrix} x_{0,0}^{hT} & x_{0,1}^{hT} & x_{0,2}^{hT} & \cdots \\ x_{0,0}^{vT} & x_{1,0}^{vT} & x_{2,0}^{vT} & \cdots \end{bmatrix}^T, \tag{4}$$

$$R_0 = \left\{ r_{0,0}, r_{0,1}, r_{0,2}, \ldots, r_{0,0}, r_{1,0}, r_{2,0}, \ldots \right\}. \tag{5}$$

We make the following assumption on the boundary condition.

Assumption 2.1: *The boundary condition is assumed to satisfy*

$$\lim_{N\to\infty} \mathbb{E}\left\{ \sum_{k=0}^{N} (\|x_{0,k}^h\|_1 + \|x_{k,0}^v\|_1) \right\} < \infty. \tag{6}$$

Next, we will introduce the definitions for positivity, asymptotic stability in the mean sense, and ℓ_1-gain in the mean sense for system (1).

Definition 2.1: System (1) is positive if $x_{i,j} \succeq 0$, $y_{i,j} \succeq 0$ for boundary condition $X_0 \succeq 0$ and disturbance $\omega_{i,j} \succeq 0$.

Definition 2.2 (Kaczorek, 2011; Liang et al., 2017): System (1) with $\omega_{i,j} \equiv 0$ is said to be asymptotically stable in the mean sense if

$$\lim_{i+j\to\infty} \mathbb{E}\left\{ \|x_{i,j}\|_1 \right\} = 0 \tag{7}$$

for boundary condition satisfying Assumption 2.1.

Definition 2.3: Given a scalar $\gamma > 0$, system (1) is said to be asymptotically stable with ℓ_1-gain γ in the mean sense if it is asymptotically stable in the mean sense and under zero boundary condition $X_0 = 0$, $\|y\|_E \le \gamma \|\omega\|_1$ for all non-zero $\omega = \{\omega_{i,j}\} \in \ell_1\{\mathbb{N}_0, \mathbb{N}_0\}$ where

$$\|y\|_E = \mathbb{E}(\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \|y_{i,j}\|_1), \ \|\omega\|_1 = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \|\omega_{i,j}\|_1. \tag{8}$$

Lemma 2.4 (Kaczorek, 2007): *The positive 2-D deterministic system*

$$\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = A \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} \tag{9}$$

is asymptotically stable if and only if there exists a vector $p \succ 0$ such that

$$p^T (A - I_u) \prec 0. \tag{10}$$

The stability of deterministic system (9) is defined as follows.

Definition 2.5: System (9) is said to be asymptotically stable if

$$\lim_{i+j\to\infty} \|x_{i,j}\|_1 = 0 \tag{11}$$

for boundary condition satisfying Assumption 2.1.

Lemma 2.6 (Zhu, Han, & Zhang, 2014): *The positive 1-D Markov jump system*

$$x_{i+1} = A(r_i)x_i, \ r_i \in \mathcal{L} \tag{12}$$

is asymptotically stable in the mean sense if and only if there exist vectors $p_m \succ 0$, $m = 1, \ldots, S$, satisfying

$$\bar{p}_m^T A_m - p_m^T \prec 0, \tag{13}$$

where $\bar{p}_m = \sum_{n=1}^{S} \pi_{mn} p_n$.

The aims of this paper are to establish

- the asymptotic stability condition for system (1);
- the ℓ_1-gain computation condition for system (1).

3. Main results

3.1. Stability analysis

Theorem 3.1: *Positive 2-D Markov jump system (1) is asymptotically stable in the mean sense if and only if there exist vectors $p_m^h \succ 0$, $p_m^v \succ 0$, $m = 1, \ldots, S$, satisfying*

$$\bar{p}_m^T A_m - p_m^T \prec 0, \tag{14}$$

where $p_m = [p_m^{hT} \ p_m^{vT}]^T$, $\bar{p}_m = \sum_{n=1}^{S} \pi_{mn} p_n$.

Proof: Define the following indicator function (Costa, Fragoso, & Marques, 2005):

$$1_{\{r(i,j)=l\}} = \begin{cases} 1, & \text{if } r_{i,j} = l,\ l \in \mathcal{L}, \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

For $(i, j) \in \mathbb{N}_0 \times \mathbb{N}_0$, $l \in \mathcal{L}$, we introduce the following notations:

$$\begin{aligned}
&\mathbf{x}^h_{i,j}(l) = \mathbb{E}\{x^h_{i,j} 1_{\{r(i,j)=l\}}\},\ \mathbf{x}^v_{i,j}(l) = \mathbb{E}\{x^v_{i,j} 1_{\{r(i,j)=l\}}\},\\
&\mathbb{X}^h_{i,j} = \begin{bmatrix} \mathbf{x}_{i,j}{}^{h\mathrm{T}}(1) & \mathbf{x}_{i,j}{}^{h\mathrm{T}}(2) & \cdots & \mathbf{x}^{h\mathrm{T}}_{i,j}(S) \end{bmatrix}^{\mathrm{T}},\\
&\mathbb{X}^v_{i,j} = \begin{bmatrix} \mathbf{x}_{i,j}{}^{v\mathrm{T}}(1) & \mathbf{x}_{i,j}{}^{v\mathrm{T}}(2) & \cdots & \mathbf{x}^{v\mathrm{T}}_{i,j}(S) \end{bmatrix}^{\mathrm{T}},\\
&\mathbb{X}_{i,j} = \begin{bmatrix} \mathbb{X}^{h\mathrm{T}}_{i,j} & \mathbb{X}^{v\mathrm{T}}_{i,j} \end{bmatrix}^{\mathrm{T}},\\
&\mathbb{X}^+_{i,j} = \begin{bmatrix} \mathbb{X}^{h\mathrm{T}}_{i+1,j} & \mathbb{X}^{v\mathrm{T}}_{i,j+1} \end{bmatrix}^{\mathrm{T}},\\
&\mathbb{Z}_{i,j} = \big[\mathbf{x}_{i,j}{}^{h\mathrm{T}}(1) \quad \mathbf{x}_{i,j}{}^{v\mathrm{T}}(1) \quad \mathbf{x}_{i,j}{}^{h\mathrm{T}}(2) \quad \mathbf{x}_{i,j}{}^{v\mathrm{T}}(2)\\
&\qquad \cdots \quad \mathbf{x}^{h\mathrm{T}}_{i,j}(S) \quad \mathbf{x}^{v\mathrm{T}}_{i,j}(S)\big]^{\mathrm{T}},\\
&\mathbb{Z}^+_{i,j} = \big[\mathbf{x}_{i+1,j}{}^{h\mathrm{T}}(1) \quad \mathbf{x}_{i,j+1}{}^{v\mathrm{T}}(1) \quad \mathbf{x}_{i+1,j}{}^{h\mathrm{T}}(2)\\
&\qquad \mathbf{x}_{i,j+1}{}^{v\mathrm{T}}(2) \cdots \quad \mathbf{x}^{h\mathrm{T}}_{i+1,j}(S) \quad \mathbf{x}^{v\mathrm{T}}_{i,j+1}(S)\big]^{\mathrm{T}},\\
&\mathbb{A} = (\Pi^{\mathrm{T}} \otimes I_u)\mathrm{diag}(A_1, A_2, \ldots, A_S).
\end{aligned} \tag{16}$$

Then there exists an orthogonal matrix

$$M = \begin{bmatrix}
I_{u_1} & 0 & 0 & 0 & 0 & \cdots & 0 & 0\\
0 & 0 & I_{u_1} & 0 & 0 & \cdots & 0 & 0\\
0 & 0 & 0 & 0 & I_{u_1} & \cdots & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & 0 & 0 & 0 & \cdots & I_{u_1} & 0\\
0 & I_{u_2} & 0 & 0 & 0 & \cdots & 0 & 0\\
0 & 0 & 0 & I_{u_2} & 0 & \cdots & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & I_{u_2}
\end{bmatrix}, \tag{17}$$

such that $\mathbb{X}_{i,j} = M\mathbb{Z}_{i,j}$. For the orthogonal matrix M, it is easy to verify that the following statements are equivalent:

- $q^{\mathrm{T}} \prec 0$;
- $q^{\mathrm{T}}M \prec 0$.

By using the above notations, for system (1) without disturbances, we can get the following equation:

$$\mathbb{Z}^+_{i,j} = \mathbb{A}\mathbb{Z}_{i,j}. \tag{18}$$

Furthermore, we have the following positive auxiliary system:

$$\mathbb{X}^+_{i,j} = M\mathbb{A}M^{\mathrm{T}}\mathbb{X}_{i,j}, \tag{19}$$

the boundary condition is obtained as

$$\begin{aligned}
\mathbb{X}_0 = \big[\mathbb{X}^{h\mathrm{T}}_{0,0} \quad \mathbb{X}^{h\mathrm{T}}_{0,1} \quad \mathbb{X}^{h\mathrm{T}}_{0,2} \quad \cdots \quad \mathbb{X}^{v\mathrm{T}}_{0,0}\\
\mathbb{X}^{v\mathrm{T}}_{1,0} \quad \mathbb{X}^{v\mathrm{T}}_{2,0} \quad \cdots\big]^{\mathrm{T}} \succeq 0.
\end{aligned} \tag{20}$$

It should be pointed out that the obtained auxiliary system (19) is a positive 2-D deterministic system.

In addition, we have

$$\begin{aligned}
\|\mathbb{X}_{i,j}\|_1 = \mathbf{1}^{\mathrm{T}}_{Su}\mathbb{X}_{i,j} &= \mathbf{1}^{\mathrm{T}}_{u_1}\sum_{l=1}^{S}\mathbf{x}^h_{i,j}(l) + \mathbf{1}^{\mathrm{T}}_{u_2}\sum_{l=1}^{S}\mathbf{x}^v_{i,j}(l)\\
&= \mathbf{1}^{\mathrm{T}}_u\mathbb{E}\{x_{i,j}\}\\
&= \mathbb{E}\{\|x_{i,j}\|_1\}.
\end{aligned} \tag{21}$$

Therefore, positive 2-D Markov jump system (1) is asymptotically stable in the mean sense if and only if auxiliary positive 2-D deterministic system (19) is asymptotically stable because $\lim_{i+j\to\infty}\|\mathbb{X}_{i,j}\|_1 = \lim_{i+j\to\infty}\mathbb{E}\{\|x_{i,j}\|_1\}$. By Lemma 2.4, auxiliary positive 2-D deterministic system (19) is asymptotically stable if and only if there exists a vector $q \succ 0$ such that

$$q^{\mathrm{T}}(M\mathbb{A}M^{\mathrm{T}} - I_{Sn}) \prec 0. \tag{22}$$

Let $p = M^{\mathrm{T}}q = [p_1^{\mathrm{T}}\ p_2^{\mathrm{T}}\ \cdots\ p_S^{\mathrm{T}}]^{\mathrm{T}}$, then the above condition can be rewritten as

$$p^{\mathrm{T}}(\mathbb{A} - I_{Su})M^{\mathrm{T}} \prec 0, \tag{23}$$

the above inequality is equivalent to

$$p^{\mathrm{T}}(\mathbb{A} - I_{Su}) \prec 0. \tag{24}$$

Substituting (16) into the above inequality yields

$$\begin{bmatrix} p_1\\ p_2\\ \vdots\\ p_S \end{bmatrix}^{\mathrm{T}} \begin{bmatrix}
\pi_{11}A_1 - I_n & \pi_{21}A_2 & \cdots & \pi_{S1}A_S\\
\pi_{12}A_1 & \pi_{22}A_2 - I_n & \cdots & \pi_{S2}A_S\\
\vdots & \vdots & \ddots & \vdots\\
\pi_{1S}A_1 & \pi_{2S}A_2 & \cdots & \pi_{SS}A_S - I_n
\end{bmatrix} \prec 0, \tag{25}$$

which is equivalent to $\bar{p}^{\mathrm{T}}_m A_m - p^{\mathrm{T}}_m \prec 0$, $\forall m \in \mathcal{L}$. Thus we can conclude that positive 2-D Markov jump system (1) is asymptotically stable in the mean sense if and only if condition (14) holds. This completes the proof. ∎

Based on Theorem 3.1 and Lemma 2.6, it is easy to obtain the following corollary.

Corollary 3.2: *Positive 2-D Markov jump system (1) is asymptotically stable in the mean sense if and only if positive 1-D Markov jump system (12) is asymptotically stable in the mean sense.*

3.2. ℓ_1-gain analysis

Theorem 3.3: *Positive 2-D Markov jump system (1) is asymptotically stable with ℓ_1-gain no greater than γ in the mean sense if there exist vectors $p_m^h \succ 0,\ p_m^v \succ 0,\ m = 1,\dots,S$, satisfying*

$$\bar{p}_m^{\mathrm{T}} A_m - p_m^{\mathrm{T}} + \mathbf{1}^{\mathrm{T}} C_m \prec 0, \tag{26}$$

$$\bar{p}_m^{\mathrm{T}} B_m + \mathbf{1}^{\mathrm{T}} D_m - \gamma \mathbf{1}^{\mathrm{T}} \prec 0, \tag{27}$$

where $p_m = [p_m^{h\mathrm{T}}\ p_m^{v\mathrm{T}}]^{\mathrm{T}}$, $\bar{p}_m = \sum_{n=1}^{S} \pi_{mn} p_n$.

Proof: On the one hand, condition (26) implies condition (14) as $C_m \succeq 0$. Then according to Theorem 1, system (1) with $\omega_{i,j} = 0$ is asymptotically stable in the mean sense.

On the other hand, for ℓ_1-gain, consider the following index:

$$\begin{aligned}\mathcal{J}_{i,j} = \mathbb{E}\Big\{&\Big[p^{h\mathrm{T}}(r_{i+1,j})x_{i+1,j}^h + p^{v\mathrm{T}}(r_{i,j+1})x_{i,j+1}^v \\ &-p^{\mathrm{T}}(r_{i,j})x_{i,j} + \mathbf{1}^{\mathrm{T}} y_{i,j}\Big] |(x_{i,j}, \omega_{i,j}, r_{i,j} = m)\Big\} \\ &- \gamma \mathbf{1}^{\mathrm{T}} \omega_{i,j},\end{aligned} \tag{28}$$

where $p(r_{i,j}) = [p^{h\mathrm{T}}(r_{i,j})\ p^{v\mathrm{T}}(r_{i,j})]^{\mathrm{T}} \succ 0$. Along the trajectory of system (1), we have

$$\begin{aligned}\mathcal{J}_{i,j} &= \bar{p}_m^{\mathrm{T}}\left(A_m x_{i,j} + B_m \omega_{i,j}\right) - p_m^{\mathrm{T}} x_{i,j} \\ &\quad + \mathbf{1}^{\mathrm{T}}\left(C_m x_{i,j} + D_m \omega_{i,j}\right) - \gamma \mathbf{1}^{\mathrm{T}} \omega_{i,j} \\ &= \left(\bar{p}_m^{\mathrm{T}} A_m - p_m^{\mathrm{T}} + \mathbf{1}^{\mathrm{T}} C_m\right) x_{i,j} \\ &\quad + \left(\bar{p}_m^{\mathrm{T}} B_m + \mathbf{1}^{\mathrm{T}} D_m - \gamma \mathbf{1}^{\mathrm{T}}\right) \omega_{i,j}.\end{aligned} \tag{29}$$

Conditions (26) and (27) ensure $\mathcal{J}_{i,j} \le 0$ for all $x_{i,j} \succeq 0,\ \omega_{i,j} \succeq 0$, and the '$=$' occurs at $x_{i,j} = 0,\ \omega_{i,j} = 0$. Based on the above relationship, we can easily obtain

$$\begin{aligned}&\mathbb{E}\left\{p^{h\mathrm{T}}(r_{0,k+1})x_{0,k+1}^h\right\} \\ &\quad = \mathbb{E}\left\{p^{h\mathrm{T}}(r_{0,k+1})x_{0,k+1}^h\right\}, \\ &\mathbb{E}\left\{p^{h\mathrm{T}}(r_{1,k})x_{1,k}^h + p^{v\mathrm{T}}(r_{0,k+1})x_{0,k+1}^v\right\} \\ &\quad \le \mathbb{E}\left\{p^{\mathrm{T}}(r_{0,k})x_{0,k} - \mathbf{1}^{\mathrm{T}} y_{0,k}\right\} + \gamma \mathbf{1}^{\mathrm{T}} \omega_{0,k}, \\ &\mathbb{E}\left\{p^{h\mathrm{T}}(r_{2,k-1})x_{2,k-1}^h + p^{v\mathrm{T}}(r_{1,k})x_{1,k}^v\right\} \\ &\quad \le \mathbb{E}\left\{p^{\mathrm{T}}(r_{1,k-1})x_{1,k-1} - \mathbf{1}^{\mathrm{T}} y_{1,k-1}\right\} + \gamma \mathbf{1}^{\mathrm{T}} \omega_{1,k-1}, \\ &\quad \vdots \\ &\mathbb{E}\left\{p^{h\mathrm{T}}(r_{k+1,0})x_{k+1,0}^h + p^{v\mathrm{T}}(r_{k,1})x_{k,1}^v\right\} \\ &\quad \le \mathbb{E}\left\{p^{\mathrm{T}}(r_{k,0})x_{k,0} - \mathbf{1}^{\mathrm{T}} y_{k,0}\right\} + \gamma \mathbf{1}^{\mathrm{T}} \omega_{k,0}, \\ &\mathbb{E}\left\{p^{v\mathrm{T}}(r_{k+1,0})x_{k+1,0}^v\right\} \\ &\quad = \mathbb{E}\left\{p^{v\mathrm{T}}(r_{k+1,0})x_{k+1,0}^v\right\}.\end{aligned} \tag{30}$$

Adding both sides of the above inequalities and considering the zero boundary conditions $x_{0,i}^h = 0,\ x_{j,0}^v = 0,\ i,\ j = 0, 1, 2, \dots$, yield

$$\begin{aligned}&\mathbb{E}\left\{\sum_{j=0}^{k+1}\left[p^{\mathrm{T}}(r_{k+1-j,j})x_{k+1-j,j}\right]\right\} \\ &\quad \le \mathbb{E}\left\{\sum_{j=0}^{k}\left[p^{\mathrm{T}}(r_{k-j,j})x_{k-j,j} - \mathbf{1}^{\mathrm{T}} y_{k-j,j}\right]\right\} \\ &\quad\quad + \gamma \sum_{j=0}^{k} \mathbf{1}^{\mathrm{T}} \omega_{k-j,j}.\end{aligned} \tag{31}$$

Taking the sum of both sides of the above inequality for k from 0 to N, we have

$$\begin{aligned}\mathbb{E}\left\{\sum_{k=0}^{N}\sum_{j=0}^{k} \mathbf{1}^{\mathrm{T}} y_{k-j,j}\right\} &\le \gamma \sum_{k=0}^{N}\sum_{j=0}^{k} \mathbf{1}^{\mathrm{T}} \omega_{k-j,j} \\ &\quad - \mathbb{E}\left\{\sum_{j=0}^{N+1}\left[p^{\mathrm{T}}(r_{N+1-j,j})x_{N+1-j,j}\right]\right\}.\end{aligned} \tag{32}$$

For $N \to \infty$, we have

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}\sum_{j=0}^{k} \mathbf{1}^{\mathrm{T}} y_{k-j,j}\right\} \le \gamma \sum_{k=0}^{\infty}\sum_{j=0}^{k} \mathbf{1}^{\mathrm{T}} \omega_{k-j,j}, \tag{33}$$

for non-zero $\omega = \{\omega_{i,j}\} \in l_1\{\mathbb{N}_0, \mathbb{N}_0\}$. The above inequality indicates $\|y\|_E \le \gamma \|\omega\|_1$. Thus we can conclude that the ℓ_1-gain of positive 2-D Markov jump system (1) in the mean sense is no greater than γ. This completes the proof. ■

3.3. Further extension to partially known transition probabilities

In some circumstances, the mode transition probabilities are not always known (Sun, Zhang, & Wu, 2018; Zhang

& Lam, 2010). Next, we will extend the asymptotic stability definition for system (1) and the stability and ℓ_1-gain analysis results to systems with partially known transition probabilities. The transition probability matrix may take the following form:

$$\Pi = \begin{bmatrix} \pi_{11} & ? & ? & \cdots & \pi_{1S} \\ ? & \pi_{22} & \pi_{23} & \cdots & ? \\ ? & ? & \pi_{33} & \cdots & \pi_{3S} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \pi_{S1} & ? & \pi_{S3} & \cdots & ? \end{bmatrix}, \quad (34)$$

where '?' represents an unknown transition probability. For $m \in \mathcal{L}$, denote

$$\mathbb{K}_m = \{n: \text{ if } \pi_{mn} \text{ is known}\}, \ \mathbb{UK}_m = \{n: \text{ if } \pi_{mn} \text{ is unknown}\}. \quad (35)$$

In the following, when dealing with the partially known probability situations, it is assumed that $\mathbb{UK}_m$ is a non-empty set.

Definition 3.4: System (1) with $\omega_{i,j} \equiv 0$ and partially known transition probabilities is said to be robustly asymptotically stable in the mean sense if

$$\lim_{i+j\to\infty} \mathbb{E}\left\{\|x_{i,j}\|_1\right\} = 0 \quad (36)$$

for boundary condition satisfying Assumption 1.

The following two theorems give the robust asymptotic stability and ℓ_1-gain computational conditions for systems with partially known transition probabilities.

Theorem 3.5: *Positive 2-D Markov jump system* (1) *with partially known transition probabilities given by (35) is robustly*
asymptotically stable in the mean sense if and only if there exist vectors $p_m^h \succ 0,\ p_m^v \succ 0,\ m = 1, \ldots, S$, *satisfying*

$$\left[\bar{p}_m^K + \left(1 - \pi_m^K\right) p_n\right]^{\mathrm{T}} A_m - p_m^{\mathrm{T}} \prec 0, \quad (37)$$

for all $n \in \mathbb{UK}_m$, *where* $p_m = [p_m^{h\mathrm{T}}\ p_m^{v\mathrm{T}}]^{\mathrm{T}}$, $\bar{p}_m^K = \sum_{n\in\mathbb{K}_m} \pi_{mn} p_n$, $\pi_m^K = \sum_{n\in\mathbb{K}_m} \pi_{mn}$.

Proof: We state with the fact that $0 \le \pi_m^K \le 1$. For $\pi_m^K = 1$, the conditions in Theorem 3.5 reduce to the conditions in Theorem 3.1. For $\pi_m^K < 1$, the left-hand side of condition (14) in Theorem 3.1 can be rewritten as

$$\begin{aligned}\Gamma_m &= \bar{p}_m^{\mathrm{T}} A_m - p_m^{\mathrm{T}} \prec 0 \\ &= \left(\bar{p}_m^K + \sum_{n\in\mathbb{UK}_m} \pi_{mn} p_n\right)^{\mathrm{T}} A_m - p_m^{\mathrm{T}} \\ &= \left[\bar{p}_m^K + \left(1-\pi_m^K\right) \sum_{n\in\mathbb{UK}_m} \frac{\pi_{mn}}{1-\pi_m^K} p_n\right]^{\mathrm{T}} A_m - p_m^{\mathrm{T}}.\end{aligned} \quad (38)$$

Since $0 \le \pi_{mn}/(1-\pi_m^K) \le 1,\ \forall n \in \mathbb{UK}_m$, and $\sum_{n\in\mathbb{UK}_m} \pi_{mn}/(1-\pi_m^K) = 1$, we have

$$\Gamma_m = \sum_{n\in\mathbb{UK}_m} \frac{\pi_{mn}}{1-\pi_m^K} \left\{\left[\bar{p}_m^K + \left(1-\pi_m^K\right) p_n\right]^{\mathrm{T}} A_m - p_m{}^{\mathrm{T}}\right\}. \quad (39)$$

Therefore, $\Gamma_m \prec 0$ is equivalent to condition (37). Then, according to Theorem 3.1, system (1) with partially known transition peobabilities in the form of (35) is robustly asymptotically stable in the mean sense if and only if conditions (37) hold. This completes the proof. ■

Theorem 3.6: *Positive 2-D Markov jump system* (1) *with partially known transition probabilities given by (35) is robustly asymptotically stable with* ℓ_1*-gain no greater than* γ *in the mean sense if there exist vectors* $p_m^h \succ 0,\ p_m^v \succ 0,\ m = 1, \ldots, S$,
satisfying

$$\left[\bar{p}_m^K + \left(1-\pi_m^K\right) p_n\right]^{\mathrm{T}} A_m - p_m^{\mathrm{T}} + \mathbf{1}^{\mathrm{T}} C_m \prec 0, \quad (40)$$

$$\left[\bar{p}_m^K + \left(1-\pi_m^K\right) p_n\right]^{\mathrm{T}} B_m + \mathbf{1}^{\mathrm{T}} D_m - \gamma \mathbf{1}^{\mathrm{T}} \prec 0, \quad (41)$$

for all $n \in \mathbb{UK}_m$, *where* $p_m = [p_m^{h\mathrm{T}}\ p_m^{v\mathrm{T}}]^{\mathrm{T}}$, $\bar{p}_m^K = \sum_{n\in\mathbb{K}_m} \pi_{mn} p_n$, $\pi_m^K = \sum_{n\in\mathbb{K}_m} \pi_{mn}$.

The proof of Theorem 3.5 can be easily obtained by a similar method presented in the proof of Theorem 3.4. We omit it here for simplicity.

If the mode transition probabilities are completely unknown, $\bar{p}_m^K$ and π_m^K are no longer present in the analysis. By removing them from inequality (36) in Theorem 3.4, we have the following two corollaries for positive 2-D switched systems with unknown transition probabilities.

Corollary 3.7: *Positive 2-D switched system* (1) *with unknown transition probabilities given by (35) is robustly* *asymptotically stable if and only if there exist vectors* $p_m^h \succ 0$, $p_m^v \succ 0$, $m = 1, \ldots, S$, *satisfying*

$$p_n^{\mathrm{T}} A_m - p_m^{\mathrm{T}} \prec 0, \tag{42}$$

for all $(m, n) \in \mathcal{L} \times \mathcal{L}$, *where* $p_m = [p_m^{h\mathrm{T}}\ p_m^{v\mathrm{T}}]^{\mathrm{T}}$.

Corollary 3.8: *Positive 2-D switched system (1) with unknown transition probabilities given by (35) is robustly asymptotically stable with* ℓ_1*-gain no greater than* γ *if there exist vectors* $p_m^h \succ 0$, $p_m^v \succ 0$, $m = 1, \ldots, S$, *satisfying*

$$p_n^{\mathrm{T}} A_m - p_m^{\mathrm{T}} + \mathbf{1}^{\mathrm{T}} C_m \prec 0, \tag{43}$$

$$p_n^{\mathrm{T}} B_m + \mathbf{1}^{\mathrm{T}} D_m - \gamma \mathbf{1}^{\mathrm{T}} \prec 0, \tag{44}$$

for all $(m, n) \in \mathcal{L} \times \mathcal{L}$, *where* $p_m = [p_m^{h\mathrm{T}}\ p_m^{v\mathrm{T}}]^{\mathrm{T}}$.

4. Illustrative example

Example 4.1: Consider system (1) with parameters given in Liang et al. (2017). The parameters of system (1) are given as follows:

$$A_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.2 & 0.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.2 & 0.6 \\ 0.5 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 0.1 & 0.0 \\ 0.1 & 0.6 \end{bmatrix},$$

$$D_1 = D_2 = \begin{bmatrix} 0.0 & 0.3 \end{bmatrix}^{\mathrm{T}}.$$

Assume that the transition matrix is given by

$$\Pi = \begin{bmatrix} 0.30 & 0.70 \\ 0.60 & 0.40 \end{bmatrix}. \tag{45}$$

Using Theorem 3.3, the obtained minimum γ is $\gamma^* = 1.3165$. The obtained value of P_1 and P_2 are

$$P_1 = \begin{bmatrix} 0.6852 \\ 1.5243 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.1101 \\ 1.4087 \end{bmatrix}.$$

Assume zero boundary conditions, and let the disturbance $\omega_{i,j}$ be

$$\omega_{i,j} = \begin{cases} 0.10, & 0 \le i \le 3,\ 0 \le j \le 10, \\ 0, & \text{otherwise.} \end{cases} \tag{46}$$

The Markov chain, trajectories of the system and the ℓ_1-gain performance are shown in Figure 1. Figure 1(a) shows the switching signal generated by the transition matrix. Figures 1(b) and (c) show the state trajectories of the system. The measured outputs are shown in Figures 1(d) and (e). The obtained ℓ_1-gain under the switching signal shown in Figure 1(a) is 0.8382. Define the following function:

$$L_{i,j} = \frac{\sum_{p=0}^{i} \sum_{q=0}^{j} \|y_{p,q}\|_1}{\sum_{p=0}^{i} \sum_{q=0}^{j} \|\omega_{p,q}\|_1}.$$

The plot of $L_{i,j}$ is shown in Figure 1(f). It is obvious that $L_{i,j}$ converges to 0.8382 as i and j increase. Running system (1) for 100 times, and the arithmetic mean of the obtained ℓ_1-gains from the 100 realisations is 0.8364, which is below the prescribed value $\gamma^* = 1.3165$.

Example 4.2: Consider system (1) with three operation modes. The parameters are given as follows:

$$A_1 = \begin{bmatrix} 0.2 & 0.5 \\ 0.2 & 0.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.2 & 0.6 \\ 0.5 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.1 & 0.3 \\ 0.4 & 0.2 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 0.1 & 0.0 \\ 0.1 & 0.6 \end{bmatrix},$$

$$D_1 = D_2 = \begin{bmatrix} 0.0 & 0.3 \end{bmatrix}^{\mathrm{T}}.$$

$$C_3 = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}.$$

The fully known transition matrix is given by

$$\Pi = \begin{bmatrix} 0.30 & 0.30 & 0.40 \\ 0.20 & 0.40 & 0.40 \\ 0.40 & 0.50 & 0.10 \end{bmatrix}. \tag{47}$$

In the following, three cases are investigated to illustrate the efficiency of the proposed theorems and corollaries.

Case 1: considering the above positive 2-D Markov jump system, where the transition probabilities are fully known.

Case 2: considering the above positive 2-D Markov jump system with partially known transition probabilities. The partially known transition matrix is given as

$$\Pi = \begin{bmatrix} 0.30 & 0.30 & 0.40 \\ 0.20 & ? & ? \\ 0.40 & 0.50 & 0.10 \end{bmatrix},$$

where the relation between known and unknown transition probabilities is

$$\pi_{22} + \pi_{23} = 1 - \pi_{21}.$$

Case 3: consider the above system with unknown transition probabilities.

Table 1. Obtained minimum ℓ_1-gain γ^* for different cases.

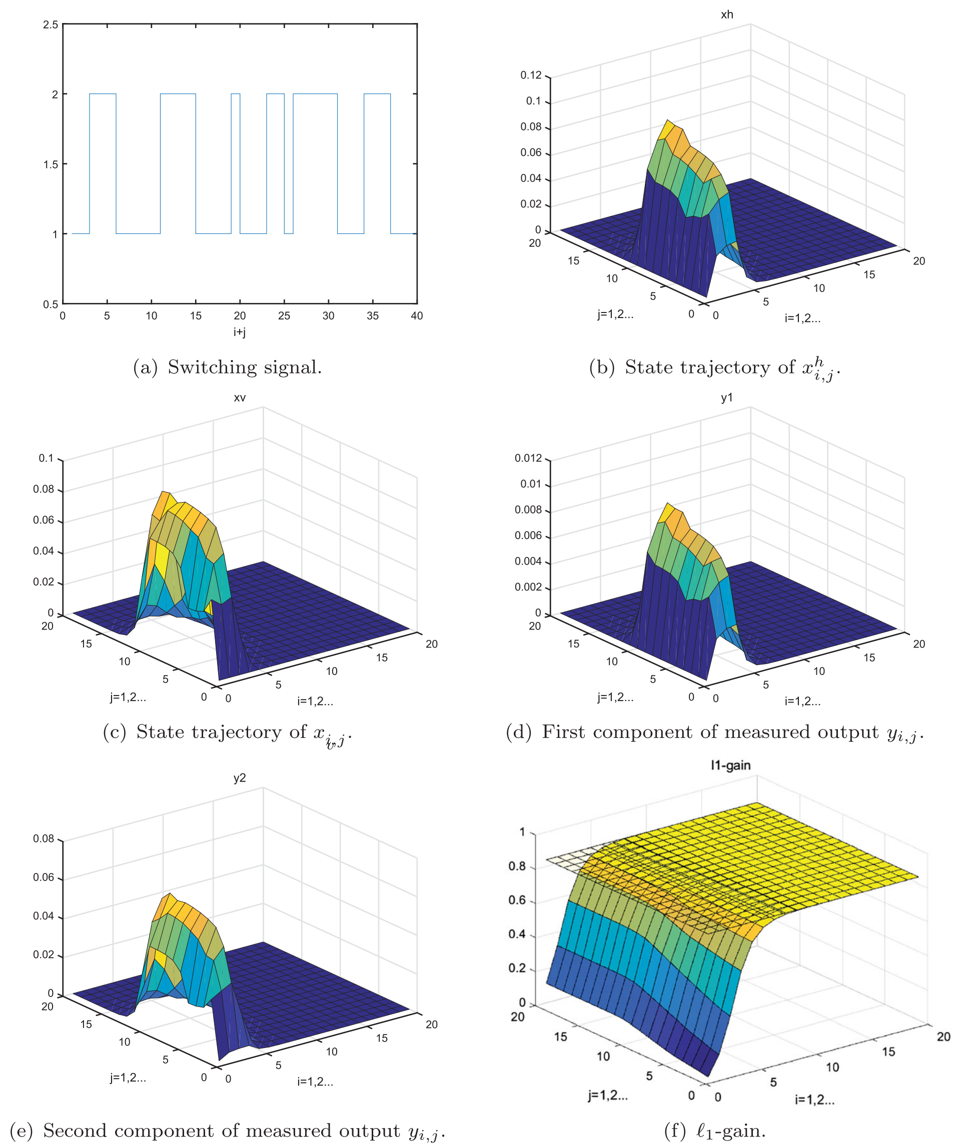
	Case 1	Case 2	Case 3
Stability analyse	Theorem 3.1 feasible	Theorem 3.3 feasible	Corollary 3.6 feasible
P_1	$\begin{bmatrix}0.6016\\1.3613\end{bmatrix}$	$\begin{bmatrix}0.6453\\1.4477\end{bmatrix}$	$\begin{bmatrix}0.9312\\1.8526\end{bmatrix}$
P_2	$\begin{bmatrix}0.9718\\1.3413\end{bmatrix}$	$\begin{bmatrix}1.1788\\1.5490\end{bmatrix}$	$\begin{bmatrix}1.3530\\1.7648\end{bmatrix}$
P_3	$\begin{bmatrix}0.8070\\1.0053\end{bmatrix}$	$\begin{bmatrix}0.8779\\1.0728\end{bmatrix}$	$\begin{bmatrix}1.2487\\1.4824\end{bmatrix}$
γ^*	1.1449	1.2336	1.5883

Using Theorems 3.3, 3.5, and Corollary 3.7, the obtained minimum ℓ_1-gain γ^* are shown in Table 1 for different cases.

From Table 1, it can be found that the more information the transition matrix has, the better the system ℓ_1-gain performance is.

5. Conclusion

The problems of stability and ℓ_1-gain analysis for positive 2-D Markov jump systems have been studied in this paper. Sufficient and necessary asymptotic stability condition and sufficient ℓ_1-gain computation condition have been derived. The obtained results are further extended to positive 2-D Markov jump systems with partially known transition probabilities. The usefulness of the obtained stability and ℓ_1-gain conditions have been verified through simulation examples.



(a) Switching signal.

(b) State trajectory of $x^h_{i,j}$.

(c) State trajectory of $x^v_{i,j}$.

(d) First component of measured output $y_{i,j}$.

(e) Second component of measured output $y_{i,j}$.

(f) ℓ_1-gain.

Figure 1. Switching signal and trajectories of the system. (a) Switching signal; (b) state trajectory of $x^h_{i,j}$; (c) state trajectory of $x^v_{i,j}$; (d) first component of measured output $y_{i,j}$; (e) second component of measured output $y_{i,j}$; (f) ℓ_1-gain.

Furthermore, our future work will extend the proposed results to the 2-D positive Markov jump systems with stochastic nonlinearities, missing measurements, and state-delays (Wang, Wang, Li, & Wang, 2016; Wei, Qiu, Karimi, & Wang, 2015).

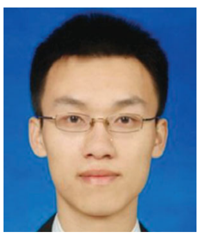
Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was partially supported by GRF HKU (17205815, 17206818), NTUT-SZU Joint Research Program (2019004) and Tencent "Rhinoceros Birds"-Scientic Research Foundation for Young Teachers of Shenzhen University.

Notes on contributors

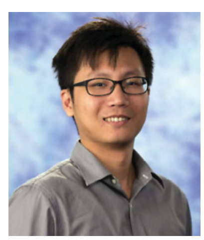


Yong Chen was born in Changsha, Hunan province, China, in 1988. He received his B.Eng. degree in Automation (ranking No. 10 among the 126 students of the major) from Harbin Institute of Technology (HIT), Harbin, Heilongjiang province, China, in 2010 and the Ph.D. degree in Mechanical Engineering from the University of Hong Kong in June, 2016. From 2016 to 2017, he was a Research Associate in the Department of Mechanical Engineering, the University of Hong Kong. Since 2018, he is an Assistant Professor in the School of Automation, Central South University, Changsha, China. His current research interests include switched systems, stochastic systems, periodic systems, fuzzy systems, positive systems in control theory and micro quadrotor in control engineering.

Chang Zhao received her B.E. degree in Detection Guidance and Control Technology from Harbin Institute of Technology, Harbin, China in July, 2015 and the Master degree in Control Science and Engineering from Harbin Institute of Technology, Harbin, China in July, 2017. She is currently working toward her Ph.D. degree in Mechanical Engineering at the University of Hong Kong. Her research interests include two dimensional systems, cyber physical systems, state estimation and robust control.

Professor J. Lam received a B.Sc. (1st Hons.) degree in Mechanical Engineering from the University of Manchester, and was awarded the Ashbury Scholarship, the A.H. Gibson Prize and the H. Wright Baker Prize for his academic performance. He obtained the M.Phil. and Ph.D. degrees from the University of Cambridge. He is a Croucher Scholar, Croucher Fellow, and Distinguished Visiting Fellow of the Royal Academy of Engineering. Prior to joining the University of Hong Kong in 1993 where he is now Chair Professor of Control Engineering, he was a lecturer at the City University of Hong Kong and the University of Melbourne. Professor Lam is a Chartered Mathematician, Chartered Scientist, Chartered Engineer, Fellow of Institute of Electrical and Electronic Engineers, Fellow of Institution of Engineering and Technology, Fellow of Institute of Mathematics and Its Applications, Fellow of Institution of Mechanical Engineers, and Fellow of Hong Kong Institution of Engineers. He is Editor-in-Chief of *IET Control Theory and Applications and Journal of The Franklin Institute*, Subject Editor of *Journal of Sound and Vibration*, Editor of *Asian Journal of Control*, Senior Editor of *Cogent Engineering*, Associate Editor of *Automatica, International Journal of Systems Science, Multidimensional Systems and Signal Processing* and *Proc. IMechE Part I: Journal of Systems and Control Engineering*. He is a member of the Engineering Panel (Joint Research Scheme), Research Grant Council, HKSAR. His research interests include model reduction, robust synthesis, delay, singular systems, stochastic systems, multidimensional systems, positive systems, networked control systems and vibration control. He is a Highly Cited Researcher in Engineering (2014, 2015, 2016, 2017, 2018) and Computer Science (2015).

Yukang Cui received his B.Eng. degree in Automation from the Harbin Institute of Technology in July, 2012 and the Ph.D. degree in Mechanical Engineering from the University of Hong Kong in March, 2017. During 2017, he was a Research Associate within the Department of Mechanical Engineering, the University of Hong Kong. Since 2018, he is an Assistant Professor in the College of Mechatronics and Control Engineering, Shenzhen University, Shenzhen, China. His current research interests include positive systems, singular systems, time-delay systems, robust control and multi-agent systems.



Ka-Wai Kwok has served as Assistant Professor in Department of Mechanical Engineering, The University of Hong Kong (HKU), since 2014. He obtained a Ph.D. at Hamlyn Centre for Robotic Surgery, Department of Computing, Imperial College London in 2012, where he continued research on surgical robotics as a postdoctoral fellow. In 2013, he was awarded the Croucher Foundation Fellowship, which supported his research jointly supervised by advisors in The University of Georgia, and Brigham and Women's Hospital – Harvard Medical School. His research interests focus on intra-operative medical image processing, surgical robotics and their control techniques. His multidisciplinary work has been recognised by various scientific communities through his international conference/journal paper awards, e.g. the Best Conference Paper Award of ICRA 2018, which is the largest conference ranked top in the field of robotics, as well as TPEL 2018, RCAR 2017, ICRA 2017, ICRA 2014, IROS 2013 and FCCM 2011, Hamlyn 2012 and 2008, and Surgical Robot Challenge 2016. He also became the recipient of the Early Career Awards 2015/16 offered by Research Grants Council (RGC) of Hong Kong.

ORCID

Chang Zhao http://orcid.org/0000-0003-4234-6547
James Lam http://orcid.org/0000-0002-0294-0640

References

Anderson, B., Agathoklis, P., Jury, E., & Mansour, M. (1986). Stability and the matrix Lyapunov equation for discrete 2-dimensional systems. *IEEE Transactions on Circuits and Systems*, *33*(3), 261–267.

Benzaouia, A., Hmamed, A., Tadeo, F., & Hajjaji, A. E. (2011). Stabilisation of discrete 2D time switched systems by state feedback control. *International Journal of Systems Science*, *42*(3), 479–487.

Chen, X., Lam, J., Li, P., & Shu, Z. (2013). ℓ_1-induced norm and controller synthesis of positive systems. *Automatica*, *49*(5), 1377–1385.

Chen, X., Lam, J., & Meng, M. (2017). Controller synthesis for positive Takagi-Sugeno fuzzy systems under ℓ_1 performance. *International Journal of Systems Science*, *48*(3), 515–524.

Costa, O. L. V., Fragoso, M. D., & Marques, R. P. (2005). *Discrete-time Markov jump linear systems*. London: Springer-Verlag.

Du, S., Qiao, J., Zhao, X., & Wang, R. (2018). Stability and ℓ_1-gain analysis for switched positive T-S fuzzy systems under asynchronous switching. *Journal of the Franklin Institute*, *355*(13), 5912–5927.

Du, C., Xie, L., & Zhang, C. (2001). H_∞ control and robust stabilization of two-dimensional systems in Roesser models. *Automatica*, *37*(2), 205–211.

Duan, Z., & Xiang, Z. (2014). H_2 output feedback controller design for discrete-time 2D switched systems. *Transactions of the Institute of Measurement and Control*, *36*(1), 68–77.

Fei, Z., Shi, S., Zhao, C., & Wu, L. (2017). Asynchronous control for 2-D switched systems with mode-dependent average dwell time. *Automatica*, *79*, 198–206.

Fornasini, E., & Marchesini, G. (1976). State-space realization theory of two-dimensional filters. *IEEE Transactions on Automatic Control*, *21*(4), 484–492.

Fornasini, E., & Marchesini, G. (1978). Double indexed dynamical systems. *Mathematical Systems Theory*, *12*, 59–72.

Fornasini, E., & Valcher, M. E. (2005). Controllability and reachability of 2-D positive systems: A graph theoretic approach. *IEEE Transactions on Circuits and Systems (I)*, *52*(3), 576–585.

Gao, H., Lam, J., Xu, S., & Wang, C. (2004). Stabilization and H_∞ control of two-dimensional Markovian jump systems. *IMA Journal of Mathematical Control and Information*, *21*(4), 377–392.

Hernandez-Vargas, E. A., Colaneri, P., & Middleton, R. H. (2014). Switching strategies to mitigate HIV mutation. *IEEE Transactions on Control Systems Technology*, *22*(4), 1623–1628.

Hinamoto, T. (1997). Stability of 2-D discrete systems described by the Fornasini-Marchesini second model. *IEEE Transactions on Circuits and Systems (I)*, *44*(3), 254–257.

Kaczorek, T. (2007). The choice of the forms of Lyapunov functions for a positive 2D Roesser model. *International Journal of Applied Mathematics and Computer Science*, *17*(4), 471–475.

Kaczorek, T. (2009). Asymptotic stability of positive 2D linear systems with delays. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, *57*(2), 133–138.

Kaczorek, T. (2011). New stability tests of positive 1D and 2D linear systems. In *Proceeding of 25th European conference modelling and simulation*, Krakow, Poland (pp. 237–243).

Kaczorek, T. (2012). *Positive 1D and 2D systems*. London: Springer-Verlag.

Kurek, J. E. (2002). Stability of positive 2-D system described by the Roesser model. *IEEE Transactions on Circuits and Systems (I)*, *49*(4), 531–533.

Li, Z., & Lam, J. (2016). Dominant pole and eigenstructure assignment for positive systems with state feedback. *International Journal of Systems Science*, *47*(12), 2901–2912.

Lian, J., Liu, J., & Zhuang, Y. (2015). Mean stability of positive Markov jump linear systems with homogeneous and switching transition probabilities. *IEEE Transactions on Circuits and Systems II: Express Briefs*, *62*(8), 801–805.

Liang, J., Pang, H., & Wang, J. (2017). Filtering and ℓ_1-gain analysis for 2-D positive system with Markov jump parameters. In *2017 11th Asian control conference*, Gold Coast, Australia (pp. 108–113).

Lu, W. S., & Lee, E. (1985). Stability analysis for two-dimensional systems via a Lyapunov approach. *IEEE Transactions on Circuits and Systems*, *32*(1), 61–68.

Roesser, R. (1975). A discrete state-space model for linear image processing. *IEEE Transactions on Automatic Control*, *20*(1), 1–10.

Shen, J., & Lam, J. (2016). Static output-feedback stabilization with optimal ℓ_1-gain for positive linear systems. *Automatica*, *63*, 248–253.

Sun, H., Zhang, Y., & Wu, A. (2018). Stochastic stability analysis of Markovian jump linear systems with incomplete transition descriptions. *IET Control Theory & Applications*, *12*(14), 1974–1982.

Wang, D., Wang, Z., Li, G., & Wang, W. (2016). Distributed filtering for switched nonlinear positive systems with missing measurements over sensor networks. *IEEE Sensors Journal*, *16*(12), 4940–4948.

Wang, J., & Zhao, J. (2017). Output tracking control with ℓ_1-gain performance for positive switched systems. *Journal of the Franklin Institute*, *354*(10), 3907–3918.

Wei, Y., Qiu, J., Karimi, H. R., & Wang, M. (2015). Model approximation for two-dimensional Markovian jump systems with state-delays and imperfect mode information. *Multidimensional Systems and Signal Processing*, *26*(3), 575–597.

Wu, L., Shi, P., Gao, H., & Wang, C. (2008). H_∞ filtering for 2D Markovian jump systems. *Automatica*, *44*(7), 1849–1858.

Wu, L., Yang, R., Shi, P., & Su, X. (2015). Stability analysis and stabilization of 2-D switched systems under arbitrary and restricted switchings. *Automatica*, *59*, 206–215.

Xiang, Z., & Huang, S. (2013). Stability analysis and stabilization of discrete-time 2D switched systems. *Circuits, Systems, and Signal Processing*, *32*(1), 401–414.

Xiang, W., Lam, J., & Shen, J. (2017). Stability analysis and ℓ_1-gain characterization for switched positive systems under dwell-time constraint. *Automatica*, *85*, 1–8.

Yang, R., Xie, L., & Zhang, C. (2006). H_2 and mixed H_2/H_∞ control of two-dimensional systems in Roesser model. *Automatica*, *42*(9), 1507–1514.

Zhang, J., Jia, X., Zhang, R., & Zuo, Y. (2018). A model predictive control framework for constrained uncertain positive systems. *International Journal of Systems Science*, *49*(4), 884–896.

Zhang, L., & Lam, J. (2010). Necessary and sufficient conditions for analysis and synthesis of Markov jump linear systems with incomplete transition descriptions. *IEEE Transactions on Automatic Control*, *55*(7), 1695–1701.

Zhu, S., Han, Q., & Zhang, C. (2014). ℓ_1-gain performance analysis and positive filter design for positive discrete-time Markov jump linear systems: A linear programming approach. *Automatica*, *50*(8), 2098–2107.

Zhu, S., Wang, B., & Zhang, C. (2017). Delay-dependent stochastic finite-time ℓ_1-gain filtering for discrete-time positive Markov jump linear systems with time-delay. *Journal of the Franklin Institute*, *354*(15), 6894–6913.