Stability and L_2 Synthesis of a Class of Periodic Piecewise Time-Varying Systems

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***Abstract*—In this paper, the stability, stabilization, and L_2-gain problems are investigated for periodic piecewise systems with time-varying subsystems. Continuous Lyapunov function with time-varying Lyapunov matrix is adopted. A condition guaranteeing the negative definiteness of a matrix polynomial, deriving from the Lyapunov derivative, is first obtained. Based on such a condition, an exponential stability condition is provided. Moreover, a state-feedback controller with time-varying gain is developed to stabilize the unstable periodic piecewise time-varying system. The L_2-gain criterion for periodic piecewise time-varying system is also studied. Numerical examples are given to show the validity of the proposed techniques.**

***Index Terms*—Controller synthesis, L_2 performance, periodic systems, stability, time-varying systems.**

I. Introduction

It is well known that periodic system is extensively present in engineering fields, such as rotor–blade system [1] and predator–prey system [2]. Moreover, periodic control also arises in a variety of applications [4], such as the vibration attenuation of the helicopter rotor–blade system, and spacecraft magnetic attitude control [5], [6]. Because of their broad application, periodic systems and periodic control play a key role in automatic control field; a lot of efforts have been put into their analysis for decades [3].

Comparing with the extensive results on discrete-time periodic systems, the analysis and synthesis on continuous-time periodic systems are more difficult [7]. In order to effectively solve the control problems of continuous-time periodic systems, numerical methods have been developed [8]–[10]. Apart from the numerical computation methods, approximation techniques have been used to help study continuous-time periodic systems. That is, the study of continuous-time periodic systems is converted to studying their approximate systems. In [11] and [12], steady-state responses of continuous-time periodic systems are estimated from periodic system with several averaged subsystems in one period, which gives rise to a periodic piecewise system. Similar techniques can be found in the recent research; as in [13], the stability result of periodic piecewise system is used to study continuous-time periodic systems in the frequency domain.

Periodic piecewise system, apart from it being treated as an approximation system, naturally has a number of applications, such as electrical circuits with ideal diodes and switches [14], mechanical systems with Coulomb friction [15], and ac–dc converters with reduced dc-link capacitance [16]. Because of its value in studying continuous-time periodic system, and its broad applications, many results have been reported on this topic [17]–[22]. Since it has switching dynamics between subsystems in one period, techniques used in switched system [23]–[29] have been adopted to investigate periodic piecewise system. Stability, stabilization, and finite-time stability analysis are reported in [17] and [18] based on the multiple Lyapunov function method. The disturbance attenuation performance such as L_2-gain, generalized H_2 indices is studied in [19] and [20] based on continuous time-varying Lyapunov functions. A saturated controller is designed in [21] to attenuate vibration of periodic piecewise system in mechanical engineering. New results on the stability and stabilization of periodic piecewise systems based on polynomial-type Lyapunov matrix are proposed in [22].

One may observe that the above-mentioned periodic piecewise system is composed of time-invariant subsystems. However, for the approximated analysis problem, using periodic piecewise time-invariant system to analyze periodic time-varying system is far from desirable, since certain dynamic properties of the original system may be lost in the approximation process. In addition, periodic piecewise system with time-varying subsystems may be more appropriate in practice, specially in power electronic equipment, such as multifunction converters [16], [30]. Unfortunately, very little results have been reported on periodic piecewise time-varying systems. This paper aims at addressing the control problems of such a system.

Specifically, the paper considers periodic piecewise time-varying systems, of which the subsystems are given in the time-interpolative form. A continuous Lyapunov function with time-varying Lyapunov matrix is adopted. A matrix polynomial definiteness problem emerges as a result of the multiplication between the time-varying Lyapunov matrix and the time-varying subsystem matrix. Different from the square matricial representation and sum of square methods [31] used in [22] to deal with the Lyapunov matrix polynomial issue, a negative defi-

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niteness condition for the matrix polynomial is proposed in this note, which utilized the time-interval information of each subsystem. Based on this condition, the exponential stability of periodic piecewise time-varying systems is studied and stabilizing controller with time-varying gain is designed. Moreover, the L_2-gain performance of periodic piecewise time-varying system is investigated as well. This paper is organized as follows. The problems studied are formulated in Section II. The stability analysis and stabilizing controller synthesis are given in Sections III and IV, respectively. L_2-gain performance is studied in Section V. Numerical examples are given in Section VI. Finally, the paper is concluded in Section VII.

Notation: $\mathbb{R}^r$ denotes the r-dimensional Euclidean space, $\mathbb{N}^+$ denotes the set of all positive integers. $\|\cdot\|$ denotes the Euclidean vector norm, the superscript $'$ refers to matrix transposition, $\overline{\lambda}(\cdot)$, $\underline{\lambda}(\cdot)$ represent the maximum, minimum eigenvalues of a real symmetric matrix, respectively. In addition, $P > 0\ (\geq 0)$ means that the matrix P is real symmetric and positive definite (positive semidefinite), $\mathcal{D}^+(\cdot)$ stands for the upper right Dini derivative, and $\mathrm{sym}(\cdot)$ denotes the sum of a matrix and its transpose matrix.

II. Problem Formulation

Consider a continuous-time periodic piecewise time-varying system given by

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + B(t)u(t) + E(t)w(t)\\ z(t) &= C(t)x(t) + D(t)w(t)\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^r$, $z(t) \in \mathbb{R}^m$, $u(t) \in \mathbb{R}^d$, $w(t) \in \mathbb{R}^l$ are the state vector, output vector, control input, and disturbance, respectively. For all $t \geq 0$, $A(t) = A(t+T)$, $B(t) = B(t+T)$, $E(t) = E(t+T)$, $C(t) = C(t+T)$, $D(t) = D(t+T)$, where T is the system fundamental period. Suppose the interval $[0, T)$ is partitioned into S subintervals $[t_{i-1}, t_i)$, $i \in \mathcal{N}$, $\mathcal{N} = \{1, 2, \ldots, S\}$, where $t_0 = 0$, $t_S = T$. In the ith subsystem, the matrices $A(t)$, $B(t)$, $C(t)$, $D(t)$, and $E(t)$ are linear time varying and denoted as $A_i(t)$, $B_i(t)$, $C_i(t)$, $D_i(t)$, and $E_i(t)$, respectively. The dwell time of the ith subsystem is $T_i = t_i - t_{i-1}$ with $\sum_{i=1}^{S} T_i = T$. In this paper, we consider $A_i(t)$ is given by, for $t \in [\ell T + t_{i-1}, \ell T + t+_i)$, $\ell = 0, 1, \ldots, i = 1, 2, \ldots, S$

$$A_i(t) = A_i + \frac{(t - \ell T - t_{i-1})}{T_i}(A_{i+1} - A_i)$$

and $B_i(t), E_i(t), C_i(t), D_i(t)$ are given in the similar interpolation formulation, with $A_i, A_{i+1}, B_i, B_{i+1}, C_i, C_{i+1}, D_i, D_{i+1}, E_i, E_{i+1}$ are constant matrices.

A definition concerning the exponential stability of system (1) is given as follows.

Definition 1: [17] The periodic piecewise system (1) with $u(t) = 0, w(t) = 0$ is said to be λ^*-exponentially stable if the solution of the system from $x(0)$ satisfies $\|x(t)\| \leq \kappa e^{-\lambda^* t}\|x(0)\|$, $\forall t > 0$ for some constants $\kappa \geq 1$, $\lambda^* > 0$.

In the following sections, we study the exponential stability, stabilizing controller synthesis, and L_2-gain performance analysis of system (1).

III. Stability Analysis

In this section, a lemma concerning the negative definiteness of a class of matrix polynomials is given. By constructing a time-varying Lyapunov matrix in linear time-interpolative form, an exponential stability condition is established based on the obtained lemma.

Consider a continuous Lyapunov function $V(x,t) = x'P(t)x$, where $P(t) > 0$ is continuous and periodic with period T. For $t \in [\ell T + t_{i-1},\ \ell T + t_i)$, $\ell = 0, 1, \ldots, i = 1, 2, \ldots, S$, $V(x,t)$ can be rewritten as

$$V(x,t) = V_i(x,t) = x'P_i(t)x \tag{2}$$

where $P(t) = P_i(t)$ for $t \in [\ell T + t_{i-1},\ \ell T + t_i)$, in other words, $\lim_{t \to \ell T + t_i} P(t) = P(\ell T + t_i)$. A general lemma concerning the stability for periodic piecewise time-varying system is given as follows.

Lemma 1: [22] Consider periodic piecewise time-varying system (1) with $u(t) = 0, w(t) = 0$, and let $\lambda^* > 0$ be a given constant. If there exist $\lambda_i, i = 1, 2, \ldots, S$, and T-periodic, continuous and Dini-differentiable matrix function $P(t)$ defined on $t \in [0, \infty)$ such that, for $t \in [\ell T + t_{i-1}, \ell T + t_i), \ell = 0, 1, \ldots, i = 1, 2, \ldots, S, P(t) = P_i(t) > 0$, satisfies

$$A_i(t)'P_i(t) + P_i(t)A_i(t) + \mathcal{D}^+ P_i(t) + \lambda_i P_i(t) < 0 \tag{3}$$

$$2\lambda^* T - \sum_{i=1}^{S} \lambda_i T_i \leq 0 \tag{4}$$

then system (1) is λ^*-exponentially stable.

In the following, we construct a T-periodic continuous time-varying Lyapunov matrix $P(t)$ such that, for $t \in [\ell T + t_{i-1}, \ell T + t_i), \ell = 0, 1, \ldots, i = 1, 2, \ldots, S$

$$\begin{aligned}P(t) &= P_i(t) = P_i + \frac{t - \ell T - t_{i-1}}{T_i}(P_{i+1} - P_i)\\ P_{S+1} &= P_1\end{aligned} \tag{5}$$

where $P_i > 0, i = 1, 2, \ldots, S$, are constant matrices and we have $0 < (\min_i \underline{\lambda}(P_i))I \leq P(t) \leq (\max_i \overline{\lambda}(P_i))I$.

Before providing the theorems, the following technique is introduced first. Let $f : [0,1]^n \to \mathbb{R}$ be a matrix polynomial function defined as

$$\begin{aligned}f(\tau_1, \tau_2, \ldots, \tau_n) = \Sigma_0 + \tau_1\Sigma_1 + \tau_1\tau_2\Sigma_2 + \cdots\\ + \left(\prod_{k=1}^{n}\tau_k\right)\Sigma_n, \quad \tau_k \in [0,1]\end{aligned} \tag{6}$$

where $n \in \mathbb{N}^+$ and $n \geq 2$, $\Sigma_j \in \mathbb{R}^{r \times r}$, $j = 0, 1, \ldots, n$, are real symmetric matrices. Thus, f is a continuous matrix function over a bounded domain, and is hence bounded. Then, one has the following lemma.

Lemma 2: Consider the matrix polynomial $f(\tau_1, \tau_2, \ldots, \tau_n)$ in (6), if

$$\sum_{k=0}^{d} \Sigma_k < 0, d = 0, 1, \ldots, n \tag{7}$$

then the matrix polynomial $f(\tau_1, \tau_2, \ldots, \tau_n) < 0$.

Proof: With $n \in \mathbb{N}^+, n \geq 2$, according to (7), one has

$$\sum_{k=0}^{n} \Sigma_k < 0, \quad \sum_{k=0}^{n-1} \Sigma_k < 0$$

then, for $0 \leq \tau_n \leq 1$, one has

$$\begin{aligned}(1 - \tau_n)(\Sigma_0 + \Sigma_1 + \cdots + \Sigma_{n-1})\\ + \tau_n(\Sigma_0 + \Sigma_1 + \cdots + \Sigma_{n-1} + \Sigma_n) < 0\end{aligned}$$

which can be rewritten as

$$\sum_{k=0}^{n-1} \Sigma_k + \tau_n\Sigma_n < 0. \tag{8}$$

With (7), one also has $\sum_{k=0}^{n-2} \Sigma_k < 0$, combining with (8), for $0 \le \tau_{n-1} \le 1$, one has

$$(1-\tau_{n-1})(\Sigma_0 + \Sigma_1 + \cdots + \Sigma_{n-2}) + \tau_{n-1}(\Sigma_0 + \cdots + \Sigma_{n-1} + \tau_n \Sigma_n) < 0$$

which can be rewritten as

$$\sum_{k=0}^{n-2} \Sigma_k + \tau_{n-1}\Sigma_{n-1} + \tau_{n-1}\tau_n \Sigma_n < 0.$$

Following the same arguments, one has

$$\Sigma_0 + \Sigma_1 + \tau_2\Sigma_2 + \tau_2\tau_3\Sigma_3 + \cdots + \left(\prod_{k=2}^{n} \tau_k\right)\Sigma_n < 0. \quad (9)$$

With (7), one also obtains $\Sigma_0 < 0$; then, combining with (9) for $0 \le \tau_1 \le 1$, one has

$$(1-\tau_1)\Sigma_0 + \tau_1\left(\Sigma_0 + \Sigma_1 + \tau_2\Sigma_2 + \cdots + \left(\prod_{k=2}^{n} \tau_k\right)\Sigma_n\right) < 0$$

which implies that $f(\tau_1, \tau_2, \ldots, \tau_n) < 0$. ∎

Remark 1: Lemma 2 is a generalization of the result in [32]. The result in [32] is valid for $n = 2$ and $\tau_1 = \tau_2$.

By exploiting Lyapunov function (2), Lyapunov matrices (5), and Lemmas 1 and 2, sufficient stability condition for periodic piecewise time-varying system (1) can be obtained in Theorem 1.

Theorem 1: Consider periodic piecewise time-varying system (1) with $u(t) = 0, w(t) = 0$. Given $\lambda^* > 0$, if there exist λ_i, and matrices $P_i > 0,\ i = 1, \ldots, S$, satisfying

$$\Sigma_{0,i} < 0 \quad (10)$$

$$\Sigma_{0,i} + \Sigma_{1,i} < 0 \quad (11)$$

$$\Sigma_{0,i} + \Sigma_{1,i} + \Sigma_{2,i} < 0 \quad (12)$$

$$P_{S+1} = P_1 \quad (13)$$

$$2\lambda^* T - \sum_{i=1}^{S} \lambda_i T_i \le 0 \quad (14)$$

where

$$\Sigma_{0,i} = A_i' P_i + P_i A_i + \frac{1}{T_i}(P_{i+1} - P_i) + \lambda_i P_i$$
$$\Sigma_{1,i} = \mathrm{sym}\left(A_i' P_{i+1} - 2A_i' P_i + A_{i+1}' P_i\right) + \lambda_i (P_{i+1} - P_i)$$
$$\Sigma_{2,i} = \mathrm{sym}\left((A_{i+1}' - A_i')(P_{i+1} - P_i)\right) \quad (15)$$

then system (1) is λ^*-exponentially stable.

Proof: For $t \in [\ell T + t_{i-1}, \ell T + t_i), \ell = 0, 1, \ldots, i = 1, 2, \ldots, S$, with (13), construct a continuous $P(t)$ as in (5). Then, one has

$$\begin{aligned}
&A_i(t)' P_i(t) + P_i(t) A_i(t) + \mathcal{D}^+ P_i(t) + \lambda_i P_i(t) \\
&= \left(A_i' + \frac{t - \ell T - t_{i-1}}{T_i}(A_{i+1}' - A_i')\right) \\
&\quad \times \left(P_i + \frac{t - \ell T - t_{i-1}}{T_i}(P_{i+1} - P_i)\right) \\
&\quad + \left(P_i + \frac{t - \ell T - t_{i-1}}{T_i}(P_{i+1} - P_i)\right) \\
&\quad \times \left(A_i + \frac{t - \ell T - t_{i-1}}{T_i}(A_{i+1} - A_i)\right) + \frac{1}{T_i}(P_{i+1} - P_i) \\
&\quad + \lambda_i \left(P_i + \frac{t - \ell T - t_{i-1}}{T_i}(P_{i+1} - P_i)\right) \\
&= A_i' P_i + P_i A_i + \frac{1}{T_i}(P_{i+1} - P_i) + \lambda_i P_i \\
&\quad + \frac{(t - \ell T - t_{i-1})}{T_i}\left(\mathrm{sym}(A_i' P_{i+1} - 2A_i' P_i + A_{i+1}' P_i)\right. \\
&\quad \left. + \lambda_i (P_{i+1} - P_i)\right) \\
&\quad + \frac{(t - \ell T - t_{i-1})^2}{T_i^2}\mathrm{sym}\left((A_{i+1}' - A_i')(P_{i+1} - P_i)\right) \\
&= \Sigma_{0,i} + \frac{(t - \ell T - t_{i-1})}{T_i}\Sigma_{1,i} + \frac{(t - \ell T - t_{i-1})^2}{T_i^2}\Sigma_{2,i}.
\end{aligned}$$

Since $t \in [\ell T + t_{i-1}, \ell T + t_i), \ell = 0, 1, \ldots, i = 1, 2, \ldots, S$, one has $0 \le \frac{t - \ell T - t_{i-1}}{T_i} \le 1$. Then, with (10)–(12) and according to Lemma 2, one has $A_i(t)' P_i(t) + P_i(t) A_i(t) + \mathcal{D}^+ P_i(t) + \lambda_i P_i(t) < 0$. Then, by combining it with (14), and according to Lemma 1, one can conclude that the periodic piecewise time-varying system is λ^*-exponentially stable. ∎

Remark 2: It can be seen that the derivative of the Lyapunov function in Theorem 1 is a matrix polynomial of degree 2. It derives from the linear interpolative formulation for both the subsystem matrices and the Lyapunov matrix. It is worth mentioning that Lemma 2 also suits for the more general case that the subsystem matrix form is given in polynomial matrix form or a Lyapunov matrix polynomial is adopted.

IV. Stabilizing Controller Synthesis

In this section, the controller with time-varying gain is designed to stabilize the unstable periodic piecewise time-varying system.

Consider a periodic time-varying state-feedback control as $u(t) = K_i(t)x(t), \quad t \in [t_{i-1}, t_i), \quad i = 1, 2, \ldots, S$, where $K_i(t)$ is continuous in the ith subsystem and $K_i(t + \ell T) = K_i(t), \quad \ell = 0, 1, \ldots$, and then the closed-loop system with $w(t) = 0$ can be obtained as

$$\dot{x}(t) = A_{ci}(t)x(t)$$
$$z(t) = C_i(t)x(t) \quad (16)$$

where $A_{ci}(t) = A_i(t) + B_i(t)K_i(t)$. Based on Lyapunov function (2) and Lyapunov matrices (5), a stabilizing controller can be obtained in Theorem 2.

Theorem 2: Consider periodic piecewise time-varying system (1) with $w(t) = 0$, and let $\lambda^* > 0$ be a given constant. If there exist λ_i and matrices $W_i > 0,\ Q_{i1},\ Q_{i,2}, i = 1, 2, \ldots, S$, satisfying

$$\Sigma_{c0,i} < 0 \quad (17)$$

$$\Sigma_{c0,i} + \Sigma_{c1,i} < 0 \quad (18)$$

$$\Sigma_{c0,i} + \Sigma_{c1,i} + \Sigma_{c2,i} < 0 \quad (19)$$

$$W_{S+1} = W_1 \quad (20)$$

$$2\lambda^* T - \sum_{i=1}^{S} \lambda_i T_i \le 0 \quad (21)$$

where

$$\begin{aligned}\Sigma_{c0,i} &= A_iW_i + W_iA_i' + B_iQ_{i,1} + Q_{i,1}'B_i' \\ &\quad - \frac{1}{T_i}(W_{i+1} - W_i) + \lambda_i W_i \\ \Sigma_{c1,i} &= \text{sym}\,(A_iW_{i+1} - 2A_iW_i + A_{i+1}W_i + B_iQ_{i,2} \\ &\quad - 2B_iQ_{i,1} + B_{i+1}Q_{i,1})) + \lambda_i(W_{i+1} - W_i) \\ \Sigma_{c2,i} &= \text{sym}\,((A_{i+1} - A_i)(W_{i+1} - W_i) \\ &\quad + (B_{i+1} - B_i)(Q_{i,2} - Q_{i,1}))\end{aligned} \tag{22}$$

then the closed-loop system is λ^*-exponentially stable, and the periodic state-feedback gain can be given as, for $t \in [\ell T + t_{i-1}, \ell T + t_i), \ell = 0, 1, \ldots, i = 1, 2, \ldots, S$, $K(t) = K_i(t) = Q_i(t)W_i^{-1}(t)$, with time-varying matrix function $Q_i(t)$ and continuous time-varying matrix function $W_i(t)$ given as

$$Q_i(t) = Q_{i,1} + \frac{t - \ell T - t_{i-1}}{T_i}(Q_{i,2} - Q_{i,1}) \tag{23}$$

$$W_i(t) = W_i + \frac{t - \ell T - t_{i-1}}{T_i}(W_{i+1} - W_i). \tag{24}$$

Proof: For $t \in [\ell T + t_{i-1}, \ell T + t_i), \ell = 0, 1, \ldots, i = 1, 2, \ldots, S$, construct $Q_i(t), W_i(t)$ as in (23) and (24). With (20), one can observe that $W(t)$ is continuous, and since $W_i > 0$, one has $W^{-1}(t) > 0$, and it is continuous. Since $0 \leq \frac{(t - \ell T - t_{i-1})}{T_i} \leq 1$, then with (17)–(19) and according to Lemma 2, one has

$$\Sigma_{c0,i} + \frac{(t - \ell T - t_{i-1})}{T_i}\Sigma_{c1,i} + \frac{(t - \ell T - t_{i-1})^2}{T_i^2}\Sigma_{c2,i} < 0 \tag{25}$$

with $Q_i(t), W_i(t)$; then (25) indicates that

$$\begin{aligned}&W_i(t)A_i'(t) + A_i(t)W_i(t) + B_i(t)Q_i(t) \\ &\quad + Q_i'(t)B_i'(t) - \mathcal{D}^+ W_i(t) + \lambda_i W_i(t) < 0.\end{aligned} \tag{26}$$

Consider a Lyapunov function $V(x,t) = x'Z(t)x$, where $Z(t) = W^{-1}(t)$. Define $Z(t) = Z_i(t)$ for $t \in [\ell T + t_{i-1}, \ell T + t_i)$, then multiplying both sides of (26) with $Z_i(t) = W_i^{-1}(t)$, and substituting $Q_i(t) = K_i(t)W_i(t)$ in (26), one has

$$\begin{aligned}&A_{ci}'W_i^{-1}(t) + W_i^{-1}(t)A_{ci} + \lambda_i W_i^{-1}(t) \\ &\quad - W_i^{-1}(t)\mathcal{D}^+ W_i(t)W_i^{-1}(t) < 0.\end{aligned} \tag{27}$$

Since $\mathcal{D}^+ W_i^{-1}(t) = -W_i^{-1}(t)\mathcal{D}^+ W_i(t)W_i^{-1}(t)$, (27) can be rewritten as

$$A_{ci}'(t)Z_i(t) + Z_i(t)A_{ci}(t) + \mathcal{D}^+ Z_i(t) + \lambda_i Z_i(t) < 0.$$

Then, combining with (21) and according to Lemma 1, the λ^*-exponential stability of the closed-loop system (1) can be established. ■

Remark 3: It is worth noticing that the constraints on λ_i in previous results have been relaxed in this note. Specifically, the sign of λ_i is not tied to the stability of the ith subsystem; the only constraint is that $\sum_{i=1}^{S} \lambda_i T_i > 2\lambda^* T$. It greatly facilitates controller design, since checking the stability of time-varying systems is generally more difficult compared with that of the time-invariant systems.

One may observe that the stabilizing controller designed in Theorem 2 is discontinuous. However, controller with continuous gain is more desirable in application. From this perspective, a stabilizing controller with continuous time-varying gain is proposed in Corollary 1, which can be easily obtained by letting $Q_{i,1} = Q_i, Q_{i,2} = Q_{i+1}, i = 1, 2, \ldots, S-1, Q_{S,2} = Q_1$ in Theorem 2.

Corollary 1: Consider periodic piecewise time-varying system (1) with $w(t) = 0$, and let $\lambda^* > 0$ be given constant. If there exist λ_i and matrices $W_i > 0,\ Q_i, i = 1, 2, \ldots, S$, satisfying (17)–(19), where

$$\begin{aligned}\Sigma_{c0,i} &= A_iW_i + W_iA_i' + B_iQ_i + Q_i'B_i \\ &\quad - \frac{1}{T_i}(W_{i+1} - W_i) + \lambda_i W_i \\ \Sigma_{c1,i} &= \text{sym}(A_iW_{i+1} - 2A_iW_i + A_{i+1}W_i + B_iQ_{i+1} \\ &\quad - 2B_iQ_i + B_{i+1}Q_i) + \lambda_i(W_{i+1} - W_i) \\ \Sigma_{c2,i} &= \text{sym}\,((A_{i+1} - A_i)(W_{i+1} - W_i) \\ &\quad + (B_{i+1} - B_i)(Q_{i+1} - Q_i))\end{aligned}$$

and $W_{S+1} = W_1, \quad Q_{S+1} = Q_1$, then the closed-loop system is λ^*-exponentially stable, and the periodic state-feedback gain can be given as, for $t \in [\ell T + t_{i-1}, \ell T + t_i), \ell = 0, 1, \ldots, i = 1, 2, \ldots, S$, $K(t) = K_i(t) = Q_i(t)W_i^{-1}(t)$, with continuous time-varying matrix function $Q_i(t)$ and $W_i(t)$ given as $Q_i(t) = Q_i + \frac{t - \ell T - t_{i-1}}{T_i}(Q_{i+1} - Q_i), W_i(t) = W_i + \frac{t - \ell T - t_{i-1}}{T_i}(W_{i+1} - W_i)$.

V. L_2-Gain Performance Analysis

In this section, the disturbance attenuation performance of the periodic piecewise time-varying system is established as an extension of the stability result.

Theorem 3: Consider periodic piecewise time-varying system (1) with $u(t) = 0$, given $\gamma > 0, \lambda^* > 0$. If there exist λ_i and matrices $P_i > 0, i = 1, 2, \ldots, S$, satisfying

$$\begin{bmatrix} \Delta_{0,i} & \Lambda_{0,i} \\ \Lambda_{0,i}' & \Omega_{0,i} \end{bmatrix} < 0 \tag{28}$$

$$\begin{bmatrix} \Delta_{0,i} & \Lambda_{0,i} \\ \Lambda_{0,i}' & \Omega_{0,i} \end{bmatrix} + \begin{bmatrix} \Delta_{1,i} & \Lambda_{1,i} \\ \Lambda_{1,i}' & \Omega_{1,i} \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} \Delta_{0,i} & \Lambda_{0,i} \\ \Lambda_{0,i}' & \Omega_{0,i} \end{bmatrix} + \begin{bmatrix} \Delta_{1,i} & \Lambda_{1,i} \\ \Lambda_{1,i}' & \Omega_{1,i} \end{bmatrix} + \begin{bmatrix} \Delta_{2,i} & \Lambda_{2,i} \\ \Lambda_{2,i}' & \Omega_{2,i} \end{bmatrix} 0 \tag{30}$$

$$P_{S+1} = P_1 \tag{31}$$

$$2\lambda^* T - \sum_{i=1}^{S} \lambda_i T_i \leq 0 \tag{32}$$

where

$$\begin{aligned}\Delta_{0,i} &= A_i'P_i + P_iA_i + C_i'C_i + \frac{1}{T_i}(P_{i+1} - P_i) + \lambda_i P_i \\ \Delta_{1,i} &= \text{sym}(A_i'P_{i+1} - 2A_i'P_i + A_{i+1}'P_i) + C_i'C_{i+1} \\ &\quad - 2C_i'C_i + C_{i+1}'C_i + \lambda_i(P_{i+1} - P_i) \\ \Delta_{2,i} &= \text{sym}\left((A_{i+1}' - A_i')(P_{i+1} - P_i)\right) \\ &\quad + (C_{i+1}' - C_i')(C_{i+1} - C_i) \\ \Lambda_{0,i} &= P_iE_i + C_i'D_i \\ \Lambda_{1,i} &= P_{i+1}E_i - 2P_iE_i + P_iE_{i+1} + C_{i+1}'D_i - 2C_i'D_i \\ &\quad + C_i'D_{i+1} \\ \Lambda_{2,i} &= (P_{i+1} - P_i)(E_{i+1} - E_i) + (C_{i+1}' - C_i')(D_{i+1} - D_i) \\ \Omega_{0,i} &= -\gamma^2 I + D_i'D_i \\ \Omega_{1,i} &= D_i'D_{i+1} - 2D_i'D_i + D_{i+1}'D_i \\ \Omega_{2,i} &= (D_{i+1}' - D_i')(D_{i+1} - D_i)\end{aligned} \tag{33}$$

then system (1) is λ^*-exponentially stable and satisfies

$$\int_0^\infty z'(\tau)z(\tau)d\tau \le aV(x_0,0) + b\gamma^2 \int_0^\infty w'(\tau)w(\tau)d\tau \qquad (34)$$

where $a = \frac{\lambda_{\max}}{2\lambda^*} e^{T\max(2\lambda^* - \lambda_{\min}, 0)}$, $\lambda_{\max} = \max_i(\lambda_i)$, and $b = \frac{\lambda_{\max}}{2\lambda^*} e^{2T\max(2\lambda^* - \lambda_{\min}, 0)}$, $\lambda_{\min} = \min_i(\lambda_i)$.

Proof: For $t \in [\ell T + t_{i-1}, \ell T + t_i)$, $\ell = 0, 1, \ldots, i = 1, 2, \ldots, S$, construct a Lyapunov function as in (2) with Lyapunov matrix given in (5).

Define $\mathcal{F} = z'z - \gamma^2 w'w$, then one has

$$\begin{aligned}
&\mathcal{D}^+ V_i(x,t) + \lambda_i V_i(x,t) + \mathcal{F} \\
&= x'(A_i'(t)P_i(t) + P_i(t)A_i(t) + \mathcal{D}^+ P(t) + \lambda_i P_i(t) \\
&\quad + C_i'(t)C_i(t))x + w'(E_i'(t)P(t) \\
&\quad + D_i'(t)C_i(t))x + x'(P_i(t)E_i(t) \\
&\quad + C_i'(t)D_i(t))w \\
&\quad + w'(-\gamma^2 I + D_i'(t)D_i(t))w \\
&= \begin{bmatrix} x \\ w \end{bmatrix}' \begin{bmatrix} \Upsilon_{0,i} & \Upsilon_{1,i} \\ \Upsilon_{1,i}' & \Upsilon_{2,i} \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix}
\end{aligned} \qquad (35)$$

where

$$\begin{aligned}
\Upsilon_{0,i} &= \Delta_{0,i} + \frac{t - \ell T - t_{i-1}}{T_i}\Delta_{1,i} + \frac{(t - \ell T - t_{i-1})^2}{T_i^2}\Delta_{2,i} \\
\Upsilon_{1,i} &= \Lambda_{0,i} + \frac{t - \ell T - t_{i-1}}{T_i}\Lambda_{1,i} + \frac{(t - \ell T - t_{i-1})^2}{T_i^2}\Lambda_{2,i} \\
\Upsilon_{2,i} &= \Omega_{0,i} + \frac{t - \ell T - t_{i-1}}{T_i}\Omega_{1,i} + \frac{(t - \ell T - t_{i-1})^2}{T_i^2}\Omega_{2,i}.
\end{aligned}$$

Thus, (35) can be rewritten as

$$\begin{aligned}
&\mathcal{D}^+ V_i(x,t) + \lambda_i V_i(x,t) + \mathcal{F} \\
&= \begin{bmatrix} x \\ w \end{bmatrix}' \left(\begin{bmatrix} \Delta_{0,i} & \Lambda_{0,i} \\ \Lambda_{0,i}' & \Omega_{0,i} \end{bmatrix} + \frac{t - \ell T - t_{i-1}}{T_i} \begin{bmatrix} \Delta_{1,i} & \Lambda_{1,i} \\ \Lambda_{1,i}' & \Omega_{1,i} \end{bmatrix} \right. \\
&\qquad \left. + \frac{(t - \ell T - t_{i-1})^2}{T_i^2} \begin{bmatrix} \Delta_{2,i} & \Lambda_{2,i} \\ \Lambda_{2,i}' & \Omega_{2,i} \end{bmatrix} \right) \begin{bmatrix} x \\ w \end{bmatrix}.
\end{aligned}$$

With $0 \le \frac{t - \ell T - t_{i-1}}{T_i} \le 1$ and (28)–(30), one has $\mathcal{D}^+ V_i(x,t) < -\lambda_i V(x,t) - \mathcal{F}$ if $x \ne 0$ or $w \ne 0$.

Integrate $\mathcal{D}^+ V_i(x,t) < -\lambda_i V(x,t) - \mathcal{F}$ for $t \in [\ell T + t_{i-1}, \ell T + t_i)$; following the similar arguments in [17], one can obtain

$$\begin{aligned}
&\sum_{k=1}^{\ell}\sum_{j=1}^{S} \int_{(k-1)T + t_{j-1}}^{(k-1)T + t_j} \exp(\Theta_1(j,k)) z'(\tau)z(\tau)d\tau \\
&\quad + \sum_{j=1}^{i-1} \int_{\ell T + t_{j-1}}^{\ell T + t_j} \exp(\Theta_2(j)) z'(\tau)z(\tau)d\tau \\
&\quad + \int_{\ell T + t_{i-1}}^{t} \exp(\Theta_3) z'(\tau)z(\tau)d\tau + V(x,t) \\
&\le \exp(\Theta_0)V(0) + \gamma^2 \left\{ \sum_{k=1}^{\ell}\sum_{j=1}^{S} \int_{(k-1)T + t_{j-1}}^{(k-1)T + t_j} \right. \\
&\quad \exp(\Theta_1(j,k)) w'(\tau)w(\tau)d\tau \\
&\quad + \sum_{j=1}^{i-1} \int_{\ell T + t_{j-1}}^{\ell T + t_j} \exp(\Theta_2(j)) w'(\tau)w(\tau)d\tau \\
&\quad \left. + \int_{\ell T + t_{i-1}}^{t} \exp(\Theta_3) w^T(\tau)w(\tau)d\tau \right\}
\end{aligned} \qquad (36)$$

where

$$\begin{aligned}
\Theta_0 &= -\ell \sum_{j=1}^{S} \lambda_j T_j - \sum_{j=1}^{i-1} \lambda_j T_j \; - \lambda_i(t - (\ell T + t_{i-1})) \\
\Theta_1(j,k) &= -\lambda_j((k-1)T + t_j - \tau) - \sum_{l=j+1}^{S} \lambda_l T_l \\
&\quad - (\ell - k)\sum_{l=1}^{S} \lambda_l T_l - \sum_{l=1}^{i-1} \lambda_l T_l - \lambda_i(t - (\ell T + t_{i-1})) \\
\Theta_2(j) &= -\lambda_j(\ell T + t_j - \tau) - \sum_{l=j+1}^{i-1} \lambda_l T_l \\
&\quad - \lambda_i(t - (\ell T + t_{i-1})) \\
\Theta_3 &= -\lambda_i(t - \tau).
\end{aligned} \qquad (37)$$

On one hand, one has

$$\begin{aligned}
\Theta_0 &\le -\ell 2\lambda^* T - \lambda_{\min} t_{i-1} - \lambda_{\min}(t - (\ell T + t_{i-1})) \\
&= -2\lambda^* t + (2\lambda^* - \lambda_{\min})(t - \ell T) \\
&< -2\lambda^* t + \max(2\lambda^* - \lambda_{\min}, 0)T \\
\Theta_1(j,k) &\le -\lambda_{\min}(kT - \tau) - (\ell - k)2\lambda^* T - \lambda_{\min}(t - \ell T) \\
&= (-\lambda_{\min} + 2\lambda^*)(t - \ell T) + 2\lambda^*(\tau - t) \\
&\quad + (-\lambda_{\min} + 2\lambda^*)(kT - \tau) \\
&\le -2\lambda^*(t - \tau) + \max(2\lambda^* - \lambda_{\min}, 0)2T \\
\Theta_2(j) &\le -2\lambda^*(t - \tau) + 2\lambda^*(t - \tau) - \lambda_{\min}(t - \tau) \\
&\le -2\lambda^*(t - \tau) + \max(2\lambda^* - \lambda_{\min}, 0)2T \\
\Theta_3 &= -2\lambda^*(t - \tau) + 2\lambda^*(t - \tau) - \lambda_m(t - \tau) \\
&\le -2\lambda^*(t - \tau) + 2\lambda^*(t - \tau) - \lambda_{\min}(t - \tau) \\
&\le -2\lambda^*(t - \tau) + \max(2\lambda^* - \lambda_{\min}, 0)2T
\end{aligned} \qquad (38)$$

On the other hand

$$\begin{aligned}
\Theta_1(j,k) &\ge -\lambda_{\max}((k-1)T + t_j - \tau) - \lambda_{\max}(T - t_j) \\
&\quad - (\ell - k)\lambda_{\max} T - \lambda_{\max} t_{i-1} - \lambda_{\max}(t - (\ell T + t_{i-1})) \\
&= -\lambda_{\max}(t - \tau)
\end{aligned} \qquad (39)$$

$$\begin{aligned}
\Theta_2(j) &\ge -\lambda_{\max}(\ell T + t_j - \tau) - \lambda_{\max}(t_{i-1} - t_j) \\
&\quad - \lambda_{\max}(t - (\ell T + t_{i-1})) \\
&= -\lambda_{\max}(t - \tau)
\end{aligned} \qquad (40)$$

$$\Theta_3 \ge -\lambda_{\max}(t - \tau). \qquad (41)$$

With $V(x,t) \ge 0$, combining (38)–(41), one has

$$\begin{aligned}
&\int_0^t e^{-\lambda_{\max}(t-\tau)} z'(\tau)z(\tau)d\tau \le e^{T(2\lambda^* - \lambda_{\min}) - 2\lambda^* t} V(x_0, 0) \\
&\quad + \gamma^2 \int_0^t e^{2T(2\lambda^* - \lambda_{\min}) - 2\lambda^*(t-\tau)} w'(\tau)w(\tau)d\tau.
\end{aligned}$$

Integrating t from 0 to ∞, one obtains (34). ■

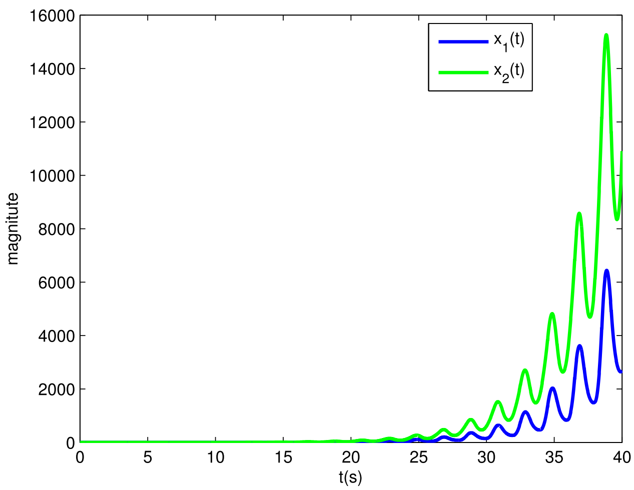

Fig. 1. Trajectory of system state.

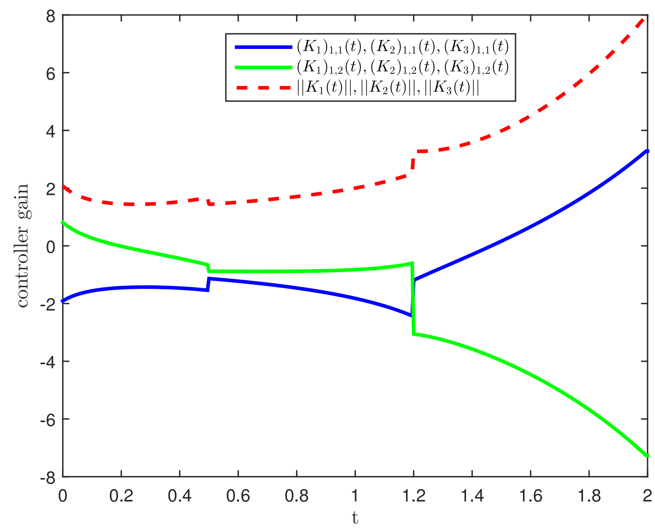

Fig. 2. Variation of $K_i(t)$ and its norm over a period.

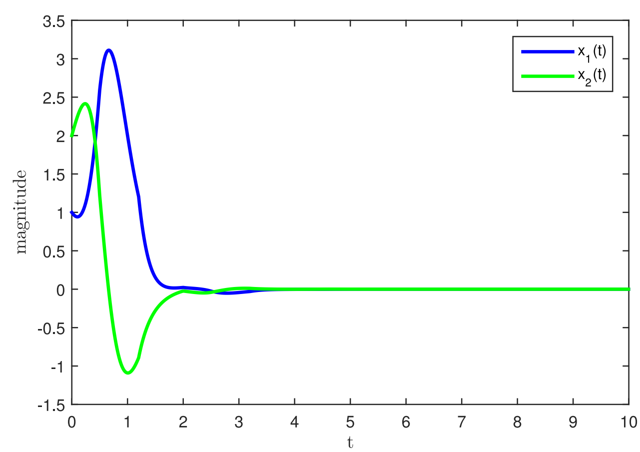

Fig. 3. Trajectory of system state under stabilizing controller.

VI. Simulation

In this section, numerical examples are used to verify the effectiveness of the proposed approaches. Example I is employed to demonstrate the merit of the proposed controller. Example II is adopted to illustrate the proposed L_2-gain performance index.

A. Example I

Consider a periodic piecewise time-varying system with $T = 2$ and $t_1 = 0.5, t_2 = 1.2, t_3 = 2$, and subsystems with $w(t) = 0$ are given as

$$A_1(t) = \begin{bmatrix} -2.1 & 0.6 \\ 0 & 1.5 \end{bmatrix} + 2(t - \ell T) \begin{bmatrix} -0.9 & 1.4 \\ 1 & -0.5 \end{bmatrix}$$

$$B_1(t) = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} + 2(t - \ell T) \begin{bmatrix} -3.5 \\ 2 \end{bmatrix}$$

$$A_2(t) = \begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix} + \frac{10}{7}(t - \ell T - 0.5) \begin{bmatrix} -1 & -1 \\ 0 & -3 \end{bmatrix}$$

$$B_2(t) = \begin{bmatrix} -3 \\ 3 \end{bmatrix} + \frac{10}{7}(t - \ell T - 0.5) \begin{bmatrix} 2 \\ -2.5 \end{bmatrix}$$

$$A_3(t) = \begin{bmatrix} -4 & 1 \\ 1 & -2 \end{bmatrix} + \frac{5}{4}(t - \ell T - 1.2) \begin{bmatrix} 1.9 & -0.4 \\ -1 & 3.5 \end{bmatrix}$$

$$B_3(t) = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix} + \frac{5}{4}(t - \ell T - 1.2) \begin{bmatrix} 1.5 \\ 0.5 \end{bmatrix}.$$

Under initial condition $x_0 = [1, 2]'$. The trajectory of the system state is shown in Fig. 1; it can be seen that the periodic piecewise time-varying system is unstable. In the following, choosing $\lambda_1 = -3, \lambda_2 = -1.5, \lambda_3 = 4$, which satisfy $\lambda_1 T_1 + \lambda_2 T_2 + \lambda_3 T_3 = 0.65 > 0$, a stabilizing controller is designed with Theorem 2. The obtained controller gain is shown in Fig. 2, and the system state under stabilizing controller is shown in Fig. 3; one can observe that system is stabilized under the proposed controller.

B. Example II

Consider a stable periodic piecewise time-varying system with $T = 2$ and $T_1 = 0.4, T_2 = 1.2, T_3 = 0.4$, and subsystems with $u(t) = 0$ are given as

$$A_1(t) = \begin{bmatrix} -4 & 1 \\ 0 & -1 \end{bmatrix} + \frac{5(t - \ell T)}{2} \begin{bmatrix} 2 & -0.6 \\ 2 & 0 \end{bmatrix}$$

$$E_1(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{5(t - \ell T)}{2} \begin{bmatrix} 0.7 \\ 0.6 \end{bmatrix}$$

$$C_1(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} + \frac{5(t - \ell T)}{2} \begin{bmatrix} 0 & -1 \end{bmatrix}, D_1(t) = 1 + 5(t - \ell T),$$

$$A_2(t) = \begin{bmatrix} -2 & 0.4 \\ 2 & -1 \end{bmatrix} + \frac{5(t - \ell T - 0.4)}{6} \begin{bmatrix} 1 & 0.1 \\ -1.5 & -1 \end{bmatrix}$$

$$E_2(t) = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix} + \frac{5(t - \ell T - 0.4)}{6} \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$$

$$C_2(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} + \frac{5(t - \ell T - 0.4)}{6} \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$$D_2(t) = 3 - \frac{5}{6}(t - \ell T - 0.4)$$

$$A_3(t) = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -2 \end{bmatrix} + \frac{5(t - \ell T - 1.6)}{2} \begin{bmatrix} -3 & 0.5 \\ -0.5 & 1 \end{bmatrix}$$

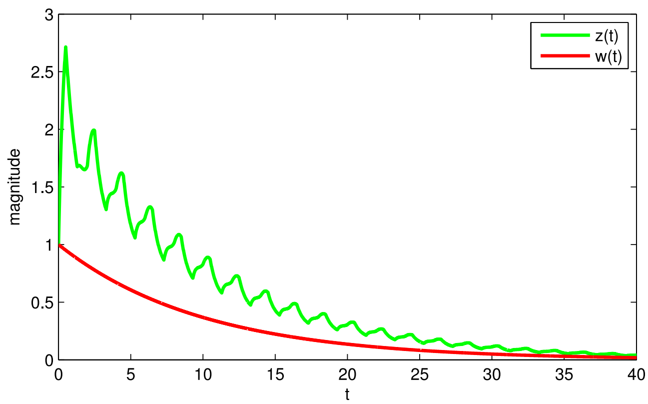

Fig. 4. Disturbance and system response.

$$E_3(t) = \begin{bmatrix} 0.5 \\ 0.6 \end{bmatrix} + \frac{5(t - \ell T - 1.6)}{2} \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}$$

$$C_3(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} + \frac{5(t - \ell T - 1.6)}{2} \begin{bmatrix} 0 & 2 \end{bmatrix}$$

$$D_3(t) = 2 - \frac{5}{2}(t - \ell T - 1.6).$$

Take $\lambda_1 = \lambda_2 = \lambda_3 = 0.2, x(0) = [0, 0]'$. According to Theorem 3, one can obtain $\gamma = 3.0386$. Consider a disturbance $w(t) = e^{-0.1t}$, then the disturbance and system output are shown in Fig. 4, and one has $\|z\|_2 = 48.7017, \|w\|_2 = 22.3718$. It can be seen that it is within the obtained L_2-gain performance index.

VII. Conclusion

In this paper, a periodic piecewise system with time-varying subsystems is considered. The negative definite characteristic of a class of matrix polynomial is studied as the basis of investigating the exponential stability condition of periodic piecewise time-varying system by employing a continuous time-varying Lyapunov matrix formulated in interpolative form. Controllers with discontinuous or continuous time-varying gain are synthesized to stabilize the system. The disturbance attenuation performance of periodic piecewise time-varying system is studied as well. Numerical examples are given to show the merits of the proposed method.

References

[1] R. H. Christensen and I. F. Santos, "Design of active controlled rotor-blade systems based on time-variant modal analysis," *J. Sound Vib.*, vol. 280, pp. 863–882, 2005.

[2] Z. Liang, X. Zeng, G. Pang, and Y. Liang, "Periodic solution of a Leslie predator-prey system with ratio-dependent and state impulsive feedback control," *Nonlinear Dyn.*, vol. 89, no. 4, pp. 2941–2955, 2017.

[3] S. Bittanti and P. Colaneri, *Periodic Systems: Filtering and Control*. London, U.K.: Springer-Verlag, 2008.

[4] B. Zhou, D. Li, and Z. Lin, "Control of discrete-time periodic linear systems with input saturation via multi-step periodic invariant sets," *Int. J. Robust Nonlinear Control*, vol. 25, pp. 103–124, 2015.

[5] S. Bittanti and F. A. Cuzzola, "Periodic active control of vibrations in helicopters: A gain-scheduled multi-objective approach," *Control Eng. Pract.*, vol. 10, no. 10, pp. 1043–1057, 2002.

[6] B. Zhou, "Global stabilization of periodic linear systems by bounded controls with applications to spacecraft magnetic attitude control," *Automatica*, vol. 60, pp. 145–154, 2015.

[7] J. Zhou, "Classification and characteristics of Floquet factorisations in linear continuous-time periodic systems," *Int. J. Control*, vol. 81, no. 11, pp. 1682–1698, 2008.

[8] P. Montagnier, R. J. Spiteri, and J. Angeles, "The control of linear time-periodic systems using Floquet-Lyapunov theory," *Int. J. Control*, vol. 77, pp. 472–490, 2004.

[9] V. Dragan and S. Aberkane, "H_2 optimal filtering for continuous-time periodic linear stochastic systems with state-dependent noise," *Syst. Control Lett.*, vol. 66, pp. 35–42, 2014.

[10] B. Zhou and G. Duan, "Periodic Lyapunov equation based approaches to the stabilization of continuous-time periodic linear systems," *IEEE Trans. Autom. Control*, vol. 57, no. 8, pp. 2139–2146, Aug. 2012.

[11] K. Farhang and A. Midha, "Steady-state response of periodic time-varying linear systems, with application to an elastic mechanism," *J. Mech. Des.*, vol. 117, pp. 633–639, 1995.

[12] T. J. Selstad and K. Farhang, "On efficient computation of the steady-state response of linear systems with periodic coefficients," *J. Vib. Acoust.*, vol. 118, no. 3, pp. 522–526, 1996.

[13] J. Zhou and H. M. Qian, "Pointwise frequency responses framework for stability analysis in periodic time-varying systems," *Int. J. Syst. Sci.*, vol. 48, no. 4, pp. 715–728, 2017.

[14] M. K. Camlibel, W. P. M. H. Heemels, A. J. van der Schaft, and J. M. Schumacher, "Switched networks and complementarity," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 50, no. 8, pp. 1036–1046, Aug. 2003.

[15] B. Acary and B. Brogliato. *Numerical Methods for Nonsmooth Dynamic Systems*. Berlin, Germany: Springer, 2008.

[16] R. Z. Scapini, L. V. Bellinaso, and L. Michels, "Stability analysis of AC-DC full-bridge converters with reduced DC-link capacitance," *IEEE Trans. Power Electron.*, vol. 33, no. 1, pp. 899–908, Jan. 2018.

[17] P. Li, J. Lam, and K. C. Cheung, "Stability, stabilization and L_2-gain analysis of periodic piecewise linear systems," *Automatica*, vol. 61, pp. 218–226, 2015.

[18] X. Xie, J. Lam, and P. Li, "Finite-time H_∞ control of periodic piecewise linear systems," *Int. J. Syst. Sci.*, vol. 48, no. 11, pp. 2333–2344, 2017.

[19] P. Li, J. Lam, Y. Chen, K. C. Cheung, and Y. Niu, "Stability and L_2-gain analysis of periodic piecewise linear systems," in *Proc. Amer. Control Conf.*, Chicago, IL, USA, Jul. 1–3, 2015, pp. 3509–3514.

[20] P. Li, J. Lam, and K. C. Cheung, "Generalized H_2 performance analysis of periodic piecewise systems," in *Proc. 34th Chin. Control Conf.*, Hangzhou, China, Jul. 28–30, 2015, pp. 77–82.

[21] P. Li, J. Lam, and K. C. Cheung, "H_∞ control of periodic piecewise vibration systems with actuator saturation," *J. Vib. Control*, vol. 23, no. 20, pp. 3377–3391, 2017.

[22] P. Li, J. Lam, K. W. Kwok, and R. Lu, "Stability and stabilization of periodic piecewise linear systems: A matrix polynomial approach," *Automatica*, vol. 94, pp. 1–8, 2018.

[23] X. Zhao, L. Zhang, P. Shi, and M. Liu, "Stability and stabilization of switched linear systems with mode-dependent average dwell time," *IEEE Trans. Autom. Control*, vol. 57, no. 7, pp. 1809–1815, Jul. 2012.

[24] G. Chesi and P. Colaneri, "Homogeneous rational Lyapunov functions for performance analysis of switched systems with arbitrary switching and dwell time constraints," *IEEE Trans. Autom. Control*, vol. 62, no. 10, pp. 5124–5137, Oct. 2017.

[25] X. Zhao, S. Yin, H. Li, and B. Niu, "Switching stabilization for a class of slowly switched systems," *IEEE Trans. Autom. Control*, vol. 60, no. 1, pp. 221–226, Jan. 2015.

[26] X. Su, X. Liu, P. Shi, and R. Yang, "Sliding mode control of discrete-time switched systems with repeated scalar nonlinearities," *IEEE Trans. Autom. Control*, vol. 62, no. 9, pp. 4604–4610, Sep. 2017.

[27] W. Xiang and J. Xiao, "Stabilization of switched continuous-time systems with all modes unstable via dwell time switching," *Automatica*, vol. 50, no. 3, pp. 940–945, 2014.

[28] W. Xiang, "Necessary and sufficient condition for stability of switched uncertain linear systems under dwell-time constraint," *IEEE Trans. Autom. Control*, vol. 61, no. 11, pp. 3619–3624, Nov. 2016.

[29] W. Xiang, H. D. Tran, and T. T. Johnson, "Output reachable set estimation for switched linear systems and its application in safety verification," *IEEE Trans. Autom. Control*, vol. 62, no. 10, pp. 5380–5387, Oct. 2017.

[30] G. Zhang *et al.*, "An impedance network boost converter with a high-voltage gain," *IEEE Trans. Power Electron.*, vol. 32, no. 9, pp. 6661–6665, Sep. 2017.

[31] G. Chesi, A. Garulli, A. Tesi, and A. Vicino, *Homogeneous Polynomial Forms for Robustness Analysis of Uncertain Systems*. London, U.K.: Springer-Verlag, 2009.

[32] C. Yang and Q. Zhang, "Multiobjective control for T-S fuzzy singularly perturbed systems," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 1, pp. 104–115, Feb. 2009.