



Brief paper

Stability and stabilization of periodic piecewise linear systems: A matrix polynomial approach[☆]

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ABSTRACT

In this paper, new conditions of stability and stabilization are proposed for periodic piecewise linear systems. A continuous Lyapunov function is constructed with a time-dependent homogeneous Lyapunov matrix polynomial. The exponential stability problem is studied first using square matricial representation and sum of squares form of homogeneous matrix polynomial. Constraints on the exponential order of each subsystem used in previous work are relaxed. State-feedback controllers with time-varying polynomial controller gain are designed to stabilize an unstable periodic piecewise system. The proposed stabilizing controller can be solved directly and effectively, which is applicable to more general situations than those previously covered. Numerical examples are given to illustrate the effectiveness of the proposed method.

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1. Introduction

Periodic linear system is a class of systems which have periodic dynamics. It is easy to find various prototypes in various engineering applications, such as rotor-bearing systems and rotor-blade systems. Much attention of periodic systems has been put on their theoretical development and engineering applications (Bittanti & Colaneri, 2008; He, Han, & Wang, 2014; Jiao, Cai, & Li, 2016; Shao & Zhao, 2017; Tao, Lu, Su, Wu, & Xu, 2017). However, one may observe that the results of control problems for continuous-time periodic systems is not that rich when compared with those of discrete-time periodic systems. This is due to that discrete-time periodic systems can be converted to time-invariant systems with the lifting techniques; while for continuous-time periodic systems, the Floquet problem which helps obtain a constant dynamic matrix is considerably more difficult to solve. Some efforts of solving the control problems for continuous-time periodic linear system can be found in Zhou (2008), Zhou and Duan (2012), Zhou, Hou, and Duan (2013) and Zhou and Qian (2017). For more results about

periodic systems, the reader may refer to Bittanti and Colaneri (2008) and its references.

Periodic piecewise linear system is a special kind of periodic linear systems, which consists of several time-invariant subsystems running periodically. Many engineering operations involving DC–DC converters and conveyor systems could be treated as periodic piecewise linear systems. From another aspect, after truncation and approximation, the periodic time-varying linear system may be described as periodic piecewise linear system (Farhang & Midha, 1995; Selstad & Farhang, 1996; Zhou & Qian, 2017). By exploiting the special dynamic characteristics, techniques targeted for periodic piecewise linear systems can be used to tackle the control problems of continuous-time periodic time-varying system. The stability of periodic time-varying system is investigated in Zhou and Qian (2017) with the periodic piecewise linear model approximation. The asymptotic stability, finite-gain L_p stability and uniformly boundedness are studied with frequency responses of the approximated periodic piecewise linear system. On the other hand, periodic piecewise linear systems can be treated as a special case of switched systems, of which the switching signal is periodic, and the switching sequence and dwell time of each subsystem is fixed. Techniques for switched systems (Xiang & Xiao, 2014; Zhai, Hu, Yasuda, & Michel, 2001; Zhao, Liu, Yin, & Li, 2014; Zhao, Zhang, Shi, & Liu, 2012) may therefore be used for periodic piecewise systems. The average dwell time approach is commonly adopted in the above results and it is also extended to the filtering problem of fuzzy switched systems with stochastic perturbation in Shi, Su, and Li (2016). The stabilization problem for discrete-time switched systems with additive disturbance is investigated in

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Zhang, Zhuang, Shi, and Zhu (2015) with quasi-time-varying Lyapunov function. It is known that the problem of time-dependent switching stabilization of switched systems composed of unstable subsystems is challenging, a novel idea of using invariant subspace analysis method to solve this issue is proposed in Zhao, Yin, and Niu (2015). With the techniques broadly used in switched systems, the exponential stability analysis of periodic piecewise linear systems can be found in Li, Lam, Chen, Cheung, and Niu (2015) and Li, Lam, and Chung (2015). A continuous time-varying Lyapunov function based on an interpolation formulation is adopted to investigate the stability problem in Li, Lam, Chen et al. (2015), for each subsystem, the Lyapunov function has its own varying rate in time. Furthermore, a discontinuous Lyapunov function is formulated in Li, Lam, and Chung (2015) to study the stability problem, which allows the Lyapunov function to have incremental bounds at the subsystem switching instants and, the Lyapunov matrix incremental bounds may be different when switching between different subsystems. It brings more slack variables and relaxes the constraints while understandably increases the computational burden and complexity. Moreover, some necessary and sufficient exponential stability conditions are also proposed in Li, Lam, and Chung (2015) based on the transition matrix, which greatly facilitates the stability verification of periodic piecewise systems. The stabilization problem of periodic piecewise system is studied in that work as well. Different controllers with constant controller gains are designed for each subsystem, and the corresponding algorithm is provided to solve the controller gain. The finite-time stability and stabilization problems of periodic piecewise are studied in Xie, Lam, and Li (2017) and a corresponding H_∞ controller is proposed in that work as well. The application of control on periodic piecewise system subject to actuator saturation can be found in Li, Lam, and Cheung (2016), where a controller is designed for periodic piecewise vibration system to attenuate the vibration.

In this work, new conditions on exponential stability and stabilization problems are investigated for periodic piecewise linear systems. Different from the previous works (Li, Lam, Chen et al., 2015; Li, Lam, & Chung, 2015), a nonlinear Lyapunov matrix polynomial is established instead of the linear interpolation Lyapunov matrix, which not only introduces more free variables but also helps relax the conditions with a more general class of Lyapunov matrices. A matrix polynomial method is used in the robustness analysis of system (Chesi, Garulli, Tesi, & Vicino, 2009), and it is also extended to switched systems with time-varying uncertainties in Briat (2015). In this work, the constructed Lyapunov matrix polynomial is continuous and time-dependent, techniques from Chesi et al. (2009) such as square matricial representation and sum of squares form are used to reformulate the Lyapunov matrix polynomial to derive the condition on exponential stability. Moreover, the constraints on some mode-dependent parameters are relaxed as well. Based on the proposed exponential stability condition, a stabilizing controller is designed as well. Comparing with the controller proposed in Li, Lam, and Chung (2015), apart from possibly lower conservatism, this controller can be solved directly with LMI conditions rather than through iterative algorithm. Moreover, controllers are also designed for unstabilizable subsystems, which may help stabilize the system more effectively and make the condition easier to solve. The paper is organized as follows. Definitions of exponential stability and matrix polynomials are provided in Section 2. In Section 3, the stability criterion for periodic piecewise linear systems with continuous time-dependent polynomial Lyapunov function are derived. Based on the result, time-varying state-feedback controllers are synthesized to stabilize the system in Section 4. Numerical examples are presented in Section 5 to demonstrate the merits of the proposed techniques and the work is concluded in Section 6.

Notation: \mathbb{R}^n denotes the n -dimensional Euclidean space, \mathbb{S}^n denotes a n -dimensional symmetric matrix. $\|\cdot\|$ stands for the

Euclidean vector norm, the superscript $'$ refers to matrix transposition, $\bar{\lambda}(\cdot)$, $\underline{\lambda}(\cdot)$ stand for the maximum, minimum eigenvalues of a real symmetric matrix, respectively. In addition, $P > 0$ means that P is a real symmetric and positive definite matrix.

2. Preliminaries

Consider a continuous-time periodic piecewise linear system of the form

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad (1)$$

where $x(t) \in \mathbb{R}^r$, $u(t) \in \mathbb{R}^s$ are the state vector and control input, respectively. It has a fundamental period T_p , that is, $A(t) = A(t + T_p)$, $B(t) = B(t + T_p)$, for $t \geq 0$. Suppose the interval $[0, T_p)$ is partitioned into S subintervals $[t_{i-1}, t_i)$, $i \in \mathcal{N}$, $\mathcal{N} = \{1, 2, \dots, S\}$, where $t_0 = 0$, $t_S = T_p$, $(A(t), B(t))$ is time-invariant under the i th subsystem and is given by (A_i, B_i) . In other words, the dwell time for the i th subsystem (A_i, B_i) is $T_i = t_i - t_{i-1}$ with $\sum_{i=1}^S T_i = T_p$. In this case, system (1) is equivalently represented by

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad t \in [\ell T_p + t_{i-1}, \ell T_p + t_i) \quad (2)$$

where $\ell = 0, 1, \dots$, $i = 1, 2, \dots, S$. A definition concerning the exponential stability of system (1) is given below.

Definition 1. Periodic piecewise system (1) with $u(t) = 0$ is said to be λ^* -exponentially stable if the solution of the system from $x(0)$ satisfies $\|x(t)\| \leq \kappa e^{-\lambda^* t} \|x(0)\|$, $\forall t \geq 0$, for some constants $\kappa \geq 1$, $\lambda^* > 0$.

In this work, a class of polynomial Lyapunov functions is adopted, some related definitions and techniques are introduced (Chesi, 2010; Chesi et al., 2009).

Definition 2 (Monomial). A function $f : \mathbb{R}^q \rightarrow \mathbb{R}$ is a monomial if

$$f(\tau) = c_a \tau^a$$

where $\tau \in \mathbb{R}^q$, $c_a \in \mathbb{R}$, $a \in \mathbb{N}^q$ and the quantity of $a_1 + \dots + a_q$ is the degree of f .

Definition 3 (Polynomial). A function $p : \mathbb{R}^q \rightarrow \mathbb{R}$ is a polynomial if

$$p(\tau) = \sum_{i=1}^q f_i(\tau)$$

where $f_i(\tau)$, $i = 1, 2, \dots, q$, is a monomial with finite degree, and the degree of p equals the largest degree of f_1, f_2, \dots, f_q . The set of all p is denoted as $\mathcal{P} = \{p : \mathbb{R}^q \rightarrow \mathbb{R}\}$, and denote the set of all p of degree h as \mathcal{P}_h .

Definition 4 (Homogeneous Polynomial). A function $p : \mathbb{R}^q \rightarrow \mathbb{R}$ is a homogeneous polynomial of degree h in q scalars if

$$p \in \mathcal{P}_h.$$

It is interesting to observe that any polynomial of degree h can be viewed as a homogeneous polynomial with one more variable set to 1. In other words, $p(\tau) = \sum_{i=0}^h p_i(\tau)$ where $p_i \in \mathcal{P}_i$ can be expressed as a homogeneous polynomial of degree i with

$$p(\tau) = \hat{p}(y)|_{y_{q+1}=1}$$

where $y = (\tau', 1)'$ and $\hat{p}(y)$ is the homogeneous polynomial given as

$$\hat{p}(y) = \sum_{i=0}^h p_i(\tau) y_{q+1}^{h-i}.$$

Definition 5 (Matrix Polynomial). The function $P : \mathbb{R}^q \rightarrow \mathbb{R}^{l \times l}$ is a matrix polynomial if

$$P_{i,j} \in \mathcal{P}, \quad i, j = 1, 2, \dots, l.$$

The set of all P is denoted as $\mathcal{P}^l = \{\Delta : \mathbb{R}^q \rightarrow \mathbb{R}^{l \times l} : P_{i,j} \in \mathcal{P}, i, j = 1, 2, \dots, l\}$.

Definition 6 (Homogeneous Matrix Polynomial). The function $P : \mathbb{R}^q \rightarrow \mathbb{R}^{l \times l}$ is a homogeneous matrix polynomial of degree h in q scalar variables if

$$P_{i,j} \in \mathcal{P}_h, \quad i, j = 1, 2, \dots, l.$$

Denote the set of all $l \times l$ homogeneous matrix polynomials of degree h in q scalar variables as \mathcal{P}_h^l .

It is worthy mentioning that any matrix polynomial P can be viewed as a homogeneous matrix polynomial with entries of P expressed as a homogeneous polynomial.

Definition 7 (Power Vector). Let $\tau^{(h)}$ be a vector in $\mathbb{R}^{\sigma(q,h)}$ such that, for all $p \in \mathcal{P}_h$, there exists $g \in \mathbb{R}^{\sigma(q,h)}$ satisfying

$$p(\tau) = g' \tau^{(h)}.$$

Then $\tau^{(h)}$ is called a power vector for \mathcal{P}_h , where

$$\sigma(q, h) = \frac{(q + h - 1)!}{(q - 1)!h!}.$$

Definition 8 (SOS (Sum of Squares) Matrix Polynomial). If there exist $P_k \in \mathcal{P}_h^l, k = 1, 2, \dots, h$, with $h \geq 1$ such that

$$P(\tau) = \sum_{k=1}^h P_k'(\tau) P_k(\tau).$$

Then, $P(\tau)$ is said to be an SOS matrix polynomial.

It is obvious that any SOS matrix polynomial is non-negative definite.

3. Stability analysis

In this section, a quadratic Lyapunov function with continuous time-dependent Lyapunov matrix polynomial is adopted to develop stability condition for periodic piecewise linear systems.

Suppose $x(t) \in \mathbb{R}^r$ and consider a Lyapunov function with the form

$$v(x, t) = x'(t)P(t)x(t), \quad (3)$$

where $P(t) \in \mathbb{R}^{r \times r}$ is chosen to be a continuous matrix polynomial dependent on t of degree $2d$, that is, for $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i), d = 1, 2, \dots, \ell = 0, 1, 2, \dots$,

$$P(t) = P_i(t) = P_{i,0} + P_{i,1}(t - \ell T_p - t_{i-1}) + \dots + P_{i,2d}(t - \ell T_p - t_{i-1})^{2d} \quad (4)$$

and $P_{i,0}, \dots, P_{i,2d}$, are constant matrices. Here $P(t)$ is a periodic time-dependent matrix polynomial with fundamental period T_p , that is, $P(t) = P(t + \ell T_p)$. To ensure continuity of $P(t)$, at each switching interval over a period, one has $P(t_1) = P(t_1^-), P(t_2) = P(t_2^-), \dots, P(t_S) = P(t_S^-)$, which lead to that $\sum_{j=0}^{2d} P_{1,j} T_1^j = P_{2,0}, \sum_{j=0}^{2d} P_{2,j} T_2^j = P_{3,0}, \dots, \sum_{j=0}^{2d} P_{S,j} T_S^j = P_{1,0}$. Then, one has $\sum_{k=1}^S \sum_{j=1}^{2d} P_{k,j} T_k^j = 0$, which implies

$$\sum_{k=1}^{S-1} \sum_{j=1}^{2d} P_{k,j} T_k^j + \sum_{j=1}^{2d-1} P_{S,j} T_S^j = -T_S^{2d} P_{S,2d}. \quad (5)$$

Since any polynomial can be viewed as a homogeneous polynomial and any matrix polynomial can be viewed as a homogeneous matrix polynomial with entries expressed as a homogeneous polynomial as mentioned before. Then, Lyapunov matrix polynomial $P(t)$ can be formulated as a homogeneous matrix polynomial given as, for $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i)$,

$$P(t) = P_i(t) = P_{i,0} 1^{2d} + P_{i,1}(t - \ell T_p - t_{i-1}) 1^{2d-1} + \dots + P_{i,2d}(t - \ell T_p - t_{i-1})^{2d}. \quad (6)$$

Define

$$\tau^{(d)}(t) = \tau_i^{(d)}(t) = (1, t - \ell T_p - t_{i-1}, \dots, (t - \ell T_p - t_{i-1})^d)' \quad (7)$$

where $\ell = 0, 1, \dots, i = 1, 2, \dots, S$, then homogeneous matrix polynomial $P(t)$ of degree $2d$ can be represented according to square matricial representation (SMR) (Chesi, Tesi, Vicino, & Genesio, 1999) via

$$P(t) = (\tau^{(d)}(t) \otimes I)' (H + L(\beta)) (\tau^{(d)}(t) \otimes I) \quad (8)$$

where $H \in \mathbb{S}^{r\sigma(2,d)}, L$ is a linear parametrization of the set

$$\mathcal{L} = \{L \in \mathbb{S}^{r\sigma(2,d)} : (\tau^{(d)}(t) \otimes I)' L(\beta) (\tau^{(d)}(t) \otimes I) = 0, \forall \tau \in \mathbb{R}^q\} \quad (9)$$

whose dimension is given as

$$\omega(2, d, r) = \frac{1}{2} r(\sigma(2, d)(r\sigma(2, d) + 1) - (r + 1)\sigma(2, 2d)).$$

It should be noticed that there are many other alternatives of choosing $\tau^{(d)}$ such as $[1, t, t^2, \dots, t^d]'$. However, the choice as (7) may be more suitable than others since it avoids expanding the term $(t - \ell T_p - t_{i-1})$ of higher degrees. Then, based on the SMR of a matrix polynomial, an SOS matrix polynomial can be constructed with the following lemma.

Lemma 1 (Chesi, 2010). A matrix polynomial P is SOS if and only if there exists β such that

$$H + L(\beta) \geq 0,$$

where H, L are given in (8).

By exploiting Lyapunov function (3), matrix polynomial (4), and Lemma 1, a sufficient condition which guarantees the exponential stability of periodic piecewise linear system is given as follows.

Theorem 1. Consider periodic piecewise linear system (2) with $u = 0$, given $\lambda^* > 0$. If there exist $\lambda_i^\diamond, i = 1, 2, \dots, S$, and real symmetric matrices $P_{i,j}$, vectors $\delta_i, \beta_i, i = 1, 2, \dots, S, j = 0, 1, \dots, 2d$, such that

$$\Psi_i(P_{i,j}) + L(\delta_i) > 0, \quad (10)$$

$$\Gamma_i(P_{i,j}) + L(\beta_i) < 0, \quad (11)$$

$$\sum_{k=1}^{S-1} \sum_{j=1}^{2d} P_{k,j} T_k^j + \sum_{j=1}^{2d-1} P_{S,j} T_S^j = -T_S^{2d} P_{S,2d}, \quad (12)$$

$$2\lambda^* T_p - \sum_{i=1}^S \lambda_i^\diamond T_i \leq 0, \quad (13)$$

where $L(\delta_i), L(\beta_i)$ are the linear parametrizations according to \mathcal{L} in (9), and $\Psi_i(P_{i,j}) \in \mathbb{S}^{r\sigma(n,d)}, \Gamma_i(\delta_{ij}) \in \mathbb{S}^{r\sigma(n,d)}$, are given as (double superscript (m, n) represents m th row, n th column block),

$$m = n = 1$$

$$\begin{aligned}
\Psi_i^{(m,n)} &= P_{1,0} + \sum_{k=1}^{i-1} \sum_{j=1}^{2d} P_{k,j} T_k^j, \\
\Gamma_i^{(m,n)} &= A_i' P_{1,0} + P_{1,0} A_i + \lambda_i^\diamond P_{1,0} + P_{i,1} \\
&\quad + \sum_{k=1}^{i-1} \sum_{j=1}^{2d} (A_i' P_{k,j} + P_{k,j} A_i + \lambda_i^\diamond P_{k,j}) T_k^j, \\
2 \leq m = n \leq d, \\
\Psi_i^{(m,n)} &= P_{i,2(m-1)}, \\
\Gamma_i^{(m,n)} &= (2m-1)P_{i,2m-1} + A_i' P_{i,2(m-1)} + P_{i,2(m-1)} A_i \\
&\quad + \lambda_i^\diamond P_{i,2(m-1)} \\
n = m = d + 1 \\
\Psi_i^{(m,n)} &= P_{i,2(m-1)}, \\
\Gamma_i^{(m,n)} &= A_i' P_{i,2d} + P_{i,2d} A_i + \lambda_i^\diamond P_{i,2d}, \\
n = m + 1, \quad 1 \leq m \leq d \\
\Psi_i^{(m,n)} &= \frac{1}{2} P_{i,2m-1}, \\
\Gamma_i^{(m,n)} &= \frac{1}{2} (2m P_{i,2m} + A_i' P_{i,2m-1} + P_{i,2m-1} A_i + \lambda_i^\diamond P_{i,2m-1}) \\
m = n + 1, \quad 1 \leq n \leq d \\
\Psi_i^{(m,n)} &= \frac{1}{2} P_{i,2n-1}, \\
\Gamma_i^{(m,n)} &= \frac{1}{2} (2n P_{i,2n} + A_i' P_{i,2n-1} + P_{i,2n-1} A_i + \lambda_i^\diamond P_{i,2n-1}),
\end{aligned}$$

elsewhere

$$0, \quad (14)$$

then system (2) is λ^* -exponentially stable.

Proof. Choose $\tau_i^{(d)}(t)$ as (7) shows, and consider Lyapunov function (3) with $P(t)$ given as (4), for $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i)$, one has

$$\begin{aligned}
v(x, t) &= x'(P_{i,0} + P_{i,1}(t - \ell T_p - t_{i-1}) + \dots \\
&\quad + P_{i,2d}(t - \ell T_p - t_{i-1})^{2d})x. \quad (15)
\end{aligned}$$

Based on the fact that $(\tau_i^{(d)}(t) \otimes I_r)' L(\delta_i)(\tau_i^{(d)}(t) \otimes I_r) = 0$, (15) can be rewritten as

$$v(x, t) = x'(\tau_i^{(d)}(t) \otimes I_r)'(\Psi_i(P_{i,j}) + L(\delta_i))(\tau_i^{(d)}(t) \otimes I_r)x.$$

With (10), we can obtain $v(x, t) > 0$ for $x \neq 0$.

Similarly, one has, for $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i)$,

$$\begin{aligned}
\dot{v}(x, t) + \lambda_i^\diamond v(x, t) &= x'(A_i' P_{i,0} + P_{i,0} A_i + P_{i,1} + \lambda_i^\diamond P_{i,0} \\
&\quad + (A_i' P_{i,1} + P_{i,1} A_i + 2P_{i,2} + \lambda_i^\diamond P_{i,1})(t - \ell T_p - t_{i-1}) + \dots \\
&\quad + (A_i' P_{i,2d-1} + P_{i,2d-1} A_i + 2dP_{i,2d} \\
&\quad + \lambda_i^\diamond P_{i,2d-1})(t - \ell T_p - t_{i-1})^{2d-1} \\
&\quad + (A_i' P_{i,2d} + P_{i,2d} A_i + \lambda_i^\diamond P_{i,2d})(t - \ell T_p - t_{i-1})^{2d})x \\
&= x'(\tau_i^{(d)}(t) \otimes I_r)'(\Gamma_i(P_{i,j}) + L(\beta_i))(\tau_i^{(d)}(t) \otimes I_r)x. \quad (16)
\end{aligned}$$

According to (11), for $x \neq 0$, one has

$$\dot{v}(x, t) < -\lambda_i^\diamond v(x, t). \quad (17)$$

Over the first period, it holds that

$$v(x, t_i) < e^{-\lambda_i^\diamond T_i} v(x, t_{i-1}). \quad (18)$$

Then with (13) we obtain

$$\begin{aligned}
v(x, t_S) &< e^{-(\sum_{i=1}^S \lambda_i^\diamond T_i - 2\lambda^* T_p + 2\lambda^* T_p)} v(x, 0) \\
&\leq e^{-2\lambda^* T_p} v(x, 0), \quad (19)
\end{aligned}$$

it is easy to conclude that

$$v(x, \ell T_p) \leq e^{-2\lambda^* T_p} v(x, (\ell - 1)T_p), \quad \ell = 1, 2, \dots, \quad (20)$$

and

$$v(x, \ell T_p) \leq e^{-2\ell T_p \lambda^*} v(x, 0), \quad (21)$$

which implies $v(x, \ell T_p) \rightarrow 0$ as $\ell \rightarrow \infty$.

Since $v(x(0), 0) = x'(0)P(0)x(0) = x'(0)P_{1,0}x(0)$ and $v(x(\ell T_p), \ell T_p) = x'(\ell T_p)P(\ell T_p)x(\ell T_p) = x'(\ell T_p)P_{1,0}x(\ell T_p)$, one can obtain that the system state satisfies

$$\|x(\ell T_p)\| \leq \nu e^{-\lambda^*(\ell T_p)} \|x(0)\|, \quad (22)$$

where $\nu = \sqrt{\bar{\lambda}(P_{1,0})/\underline{\lambda}(P_{1,0})}$.

Consider that

$$x(t) = e^{A_i(t - \ell T_p - t_{i-1})} x(t_{i-1}), \quad t \in [\ell T_p + t_{i-1}, \ell T_p + t_i), \quad (23)$$

and follow the steps in Li, Lam, and Chung (2015), then one has

$$\begin{aligned}
\|x(t)\| &\leq \eta \|x(\ell - 1)T_p\| \leq \beta \nu e^{\frac{2\lambda^*(\ell-1)T_p}{2}} \|x(0)\| \\
&= \eta \nu e^{\lambda^* T_p} e^{-\lambda^* \ell T_p} \|x(0)\|, \quad (24)
\end{aligned}$$

where $\eta = \Gamma_{i=1}^S \phi_i$. Since $t < \ell T_p$, it implies $e^{-\lambda^* t} > e^{-\lambda^* \ell T_p}$. Hence,

$$\|x(t)\| \leq \kappa e^{-\lambda^* t} \|x(0)\|, \quad (25)$$

where $\kappa = \beta \nu e^{\lambda^* T_p}$. Therefore, the periodic system is λ^* -exponentially stable. \square

Remark 1. Due to the continuity of Lyapunov function, there is an equality constraint (12) in Theorem 1. It should be noticed that with this equality, from (5), for the S th subsystem, one has

$$\begin{aligned}
\Psi_S^{(d+1,d+1)} &= -\frac{1}{T_S^{2d}} \left(\sum_{k=1}^{S-1} \sum_{j=1}^{2d} P_{k,j} T_k^j + \sum_{j=1}^{2d-1} P_{S,j} T_S^j \right), \\
\Gamma_S^{(d,d+1)} &= \Gamma_S^{(d+1,d)} \\
&= \frac{1}{2} (A_S' P_{S,2n-1} + P_{S,2n-1} A_S + \lambda_d^\diamond P_{S,2n-1}) \\
&\quad - \frac{d}{T_S^{2d}} \left(\sum_{k=1}^{S-1} \sum_{j=1}^{2d} P_{k,j} T_k^j + \sum_{j=1}^{2d-1} P_{S,j} T_S^j \right), \\
\Gamma_S^{(d+1,d+1)} &= -\frac{1}{T_S^{2d}} A_S' \left(\sum_{k=1}^{S-1} \sum_{j=1}^{2d} P_{k,j} T_k^j + \sum_{j=1}^{2d-1} P_{S,j} T_S^j \right) \\
&\quad - \frac{1}{T_S^{2d}} \left(\sum_{k=1}^{S-1} \sum_{j=1}^{2d} P_{k,j} T_k^j + \sum_{j=1}^{2d-1} P_{S,j} T_S^j \right) A_S \\
&\quad - \frac{\lambda_S^\diamond}{T_S^{2d}} \left(\sum_{k=1}^{S-1} \sum_{j=1}^{2d} P_{k,j} T_k^j + \sum_{j=1}^{2d-1} P_{S,j} T_S^j \right). \quad \square
\end{aligned}$$

Remark 2. In Li, Lam, and Chung (2015), constraints were imposed on the parameter λ_i^\diamond for each subsystem. Specifically, for a Hurwitz subsystem, positive λ_i^\diamond should be imposed, while for a non-Hurwitz subsystem, negative λ_i^\diamond was imposed. In this method, such constraints are relaxed, λ_i^\diamond is only needed to satisfy $2\lambda^* T_p - \sum_{i=1}^S \lambda_i^\diamond T_i \leq 0$ over a period. In other words, negative λ_i^\diamond may be associated with Hurwitz subsystems and positive λ_i^\diamond may possibly be associated with non-Hurwitz subsystems. It introduces more freedom to solve the condition. \square

It is worth mentioning that to employ matrix polynomials, their degrees should be even and greater than or equal to 2, the condition

proposed in Theorem 1 cannot be reduced to the interpolation case as in Li, Lam, Chen et al. (2015) with $P_{i,j} = 0, i = 1, 2, \dots, S, j = 2, 3, \dots, 2d$.

4. Stabilizing controller synthesis

In this section, controllers with time-varying gains are designed to stabilize the unstable periodic piecewise linear system with possibly unstabilizable subsystems.

Consider a periodic time-varying state-feedback control as

$$u(t) = K_i(t)x(t), \quad t \in [\ell T_p + t_{i-1}, \ell T_p + t_i),$$

$$i = 1, 2, \dots, S, \quad \ell = 0, 1, \dots \quad (26)$$

where $K_i(t)$ is continuous in the i th subsystem and $K_i(t + \ell T_p) = K_i(t)$.

With controller (26), the closed-loop form of system (2) can be obtained as

$$\dot{x} = A_{ci}(t)x(t) \quad (27)$$

where $A_{ci}(t) = A_i + B_i K_i(t)$.

It should be noticed that, differently from open-loop system (1) which includes time-invariant subsystems, the closed-loop system consists of time-varying subsystem because of the time-varying controllers. To facilitate the development, introduce the Dini derivative (Garg, 1998) of a continuous function $Z(t)$ given by

$$D^+ Z(t) = D^+ Z_i(t) = \limsup_{h \rightarrow 0^+} \frac{Z_i(t+h) - Z_i(t)}{h},$$

$$t \in [\ell T_p + t_{i-1}, \ell T_p + t_i) \quad (28)$$

then one can obtain a stability result for the time-varying subsystem case as follows.

Theorem 2. Consider periodic piecewise system (27), let $\lambda^* > 0$, be a given constant. If there exist $\lambda_i^\diamond, i = 1, 2, \dots, S$, and a real symmetric T_p -periodic, continuous and Dini-differentiable matrix function $Z(t)$ defined on $t \in [0, \infty)$ such that, for $i = 1, 2, \dots, S, t \in [\ell T_p + t_{i-1}, \ell T_p + t_i), \ell = 1, 2, \dots, Z(t) = Z_i(t)$ satisfies

$$A'_{ci}(t)Z_i(t) + Z_i(t)A_{ci}(t) + D^+ Z_i(t) + \lambda_i^\diamond Z_i(t) < 0, \quad (29)$$

$$2\lambda^* T_p - \sum_{i=1}^S \lambda_i^\diamond T_i \leq 0, \quad (30)$$

then system (27) is λ^* -exponentially stable.

Proof. Construct a periodic Lyapunov function, for $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i), \ell = 0, 1, \dots, i = 1, 2, \dots, S$,

$$v(x, t) = v_i(x, t) = x^T(t)Z(t)x(t) = x^T(t)Z_i(t)x(t)$$

where $Z(t) > 0$ is continuous. It is obvious that $v(t) > 0$ for $x \neq 0$. By employing the Dini derivative of $Z(t)$ as (28), then one can obtain that

$$D^+ v_i(x, t) + \lambda_i^\diamond v_i(x, t) = A'_{ci}(t)Z_i(t) + Z_i(t)A_{ci}(t) + D^+ Z_i(t) + \lambda_i^\diamond Z_i(t), \quad (31)$$

with (29), one has $D^+ v_i(x, t) < -\lambda_i^\diamond v_i(x, t)$. Then, by following the steps of the proof for Theorem 1, one has

$$\|x(\ell T_p)\| \leq v' e^{-\lambda^*(\ell T_p)} \|x(0)\|, \quad (32)$$

where $v' = \sqrt{\bar{\lambda}(Z(0))/\underline{\lambda}(Z(0))}$.

According to Coppel's inequality (Hewer, 1974), one has

$$\|x(t)\| \leq e^{\int_{\ell T_p + t_{i-1}}^t \mu(A_{ci}(\tau)) d\tau} \|x(\ell T_p + t_{i-1})\|$$

where $\mu(A_{ci}(\tau)) = \frac{1}{2} \bar{\lambda}(A_{ci}(\tau) + A'_{ci}(\tau))$. Since $\tau \in [\ell T_p + t_{i-1}, t)$, then $\mu(A_{ci}(\tau))$ is continuous and bounded, hence there exists a constant $\hat{\mu}_i$ such that $\hat{\mu}_i \geq \mu(A_{ci}(\tau))$, then one has

$$\|x(t)\| \leq e^{\int_{\ell T_p + t_{i-1}}^t \frac{1}{2} \bar{\lambda}(A_{ci}(\tau) + A'_{ci}(\tau)) d\tau} \|x(\ell T_p + t_{i-1})\|$$

$$\leq e^{\hat{\mu}_i(t - \ell T_p - t_{i-1})} \|x(\ell T_p + t_{i-1})\|$$

$$\leq \max(1, e^{\hat{\mu}_i T_i}) \|x(\ell T_p + t_{i-1})\|.$$

Similar to the steps in Li, Lam, Chen et al. (2015), one has

$$\|x(t)\| \leq \beta' \|x(\ell - 1)T_p\| \leq \beta v' e^{\lambda^*(\ell - 1)T_p} \|x(0)\|$$

$$= \beta' v' e^{\lambda^* T_p} e^{-\lambda^* \ell T_p} \|x(0)\| \quad (33)$$

where $\beta' = \prod_{i=1}^S \max(1, e^{\hat{\mu}_i T_i})$. Since $t < \ell T_p$, it implies $e^{-\lambda^* t} > e^{-\lambda^* \ell T_p}$. Hence, one has

$$\|x(t)\| \leq \kappa' e^{-\lambda^* t} \|x(0)\| \quad (34)$$

where $\kappa' = \beta' v' \lambda^* T_p$. Therefore, system (27) is λ^* -exponentially stable. \square

Then, based on Theorem 2, then the existence conditions of a set of stabilizing controllers of periodic piecewise linear system can be given as follows.

Theorem 3. Consider periodic piecewise system (2), let $\lambda^* > 0$ be a given constant. If there exist $\lambda_i^\diamond, i = 1, 2, \dots, S$, and matrices $W_{i,j}, Q_{i,k}$, vectors $\varepsilon_{ci}, \zeta_{ci}, i = 1, 2, \dots, S, j = 1, 2, \dots, 2d$, such that

$$\Psi_{ci}(W_{i,j}) + L(\varepsilon_{ci}) > 0, \quad (35)$$

$$\Gamma_{ci}(W_{i,j}, Q_{i,j}) + L(\zeta_{ci}) < 0, \quad (36)$$

$$\sum_{k=1}^{S-1} \sum_{j=1}^{2d} W_{k,j} T_k^j + \sum_{j=1}^{2d-1} W_{S,j} T_S^j = -T_S^{2d} W_{S,2d}, \quad (37)$$

$$2\lambda^* T_p - \sum_{i=1}^S \lambda_i^\diamond T_i \leq 0, \quad (38)$$

where $L(\varepsilon_{ci}), L(\zeta_{ci})$ are the linear parametrizations according to \mathcal{L} in (9) and $\Psi_{ci}(W_{i,j}) \in \mathbb{S}^{r\sigma(n,d)}, \Gamma_{ci}(W_{i,j}, Q_{i,j}) \in \mathbb{S}^{r\sigma(n,d)}$, are given as (superscript (m, n) represents the m th row, n th column block)

$$m = n = 1$$

$$\Psi_{ci}^{(m,n)} = W_{1,0} + \sum_{k=1}^{i-1} \sum_{j=1}^{2d} W_{k,j} T_k^j,$$

$$\Gamma_{ci}^{(m,n)} = W_{1,0} A'_i + A_i W_{1,0} + \lambda_i^\diamond W_{1,0} - W_{i,1}$$

$$+ \sum_{k=1}^{i-1} \sum_{j=1}^{2d} (W_{k,j} A'_i + A_i W_{k,j} + \lambda_i^\diamond W_{k,j}) T_k^j,$$

$$+ Q'_{i,0} B'_i + B_i Q_{i,0}$$

$$2 \leq m = n \leq d,$$

$$\Psi_{ci}^{(m,n)} = W_{i,2(m-1)},$$

$$\Gamma_{ci}^{(m,n)} = -(2m-1)W_{i,2m-1} + W_{i,2(m-1)} A'_i + A_i W_{i,2(m-1)}$$

$$+ Q'_{i,2(m-1)} B'_i + B_i Q_{i,2(m-1)} + \lambda_i^\diamond W_{i,2(m-1)},$$

$$n = m = d + 1$$

$$\Psi_{ci}^{(m,n)} = W_{i,2(m-1)},$$

$$\Gamma_{ci}^{(m,n)} = W_{i,2d} A'_i + A_i W_{i,2d} + Q'_{i,2d} B'_i$$

$$+ B_i Q_{i,2d} + \lambda_i^\diamond W_{i,2d},$$

$$n = m + 1, \quad 1 \leq m \leq d$$

$$\Psi_{ci}^{(m,n)} = \frac{1}{2} W_{i,2m-1},$$

$$\Gamma_{ci}^{(m,n)} = \frac{1}{2} (-2m W_{i,2m} + W_{i,2m-1} A'_i + A_i W_{i,2m-1})$$

$$\begin{aligned}
& + Q'_{i,2m-1}B'_i + B_iQ_{i,2m-1} + \lambda_i^\diamond W_{i,2m-1}), \\
m & = n + 1, \quad 1 \leq n \leq d \\
\Psi_i^{(m,n)} & = \frac{1}{2}W_{i,2n-1}, \\
\Gamma_i^{(m,n)} & = \frac{1}{2}(-2nW_{i,2n} + W_{i,2n-1}A'_i + A_iW_{i,2n-1} \\
& + Q'_{i,2n-1}B'_i + B_iQ_{i,2n-1} + \lambda_i^\diamond W_{i,2n-1}), \\
\text{elsewhere} & \\
0, & \tag{39}
\end{aligned}$$

then the closed-loop system is λ^* -exponentially stable, the periodic state-feedback gain can be given as for $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i)$,

$$K(t) = K_i(t) = Q_i(t)W^{-1}(t), \tag{40}$$

with time-varying matrix function $Q_i(t)$ and continuous time-varying matrix function $W(t)$ given in polynomial forms as

$$\begin{aligned}
Q_i(t) & = Q_{i,0} + Q_{i,1}(t - \ell T_p - t_{i-1}) + \dots \\
& + Q_{i,2d}(t - \ell T_p - t_{i-1})^{2d}, \quad i = 1, 2, \dots, S \\
i & = 1 \\
W(t) & = W_1 = W_{1,0} + W_{1,1}(t - \ell T_p) + \dots + W_{1,2d}(t - \ell T_p)^{2d} \\
i & = 2, \dots, S - 1 \\
W(t) & = W_i = W_{i,0} + \sum_{k=1}^{i-1} \sum_{j=1}^{2d} W_{k,j}T_k^j + W_{i,1}(t - \ell T_p - t_{i-1}) \\
& + \dots + W_{i,2d}(t - \ell T_p - t_{i-1})^{2d} \\
i & = S \\
W(t) & = W_S = W_{1,0} + \sum_{k=1}^{S-1} \sum_{j=1}^{2d} W_{k,j}T_k^j + W_{S,1}(t - \ell T_p - t_{S-1}) \\
& + \dots + W_{S,2d-1}(t - \ell T_p - t_{S-1}) \\
& - \frac{1}{T_S^{2d}} \left(\sum_{k=1}^{S-1} \sum_{j=1}^{2d} W_{k,j}T_S^j + \sum_{j=1}^{2d-1} W_{S,j}T_S^j \right)
\end{aligned}$$

Proof. For $t \in [\ell T_p + t_{i-1}, \ell T_p + t_i)$, $i = 1, 2, \dots, S$, $\ell = 1, 2, \dots$, based on (35) and the matrix function of $W(t)$, one has

$$W^{-1}(t) > 0,$$

and it is continuous. Construct a Lyapunov function $v = x'(t)W^{-1}(t)x(t) = x'(t)Z(t)x(t)$, then one has $Z(t)$ continuous, $Z(t) > 0$, and $v(t) > 0$ for $x(t) \neq 0$.

On the other hand, based on $(\tau_i^{(d)} \otimes I_r)'L(\zeta_i)(\tau_i^{(d)} \otimes I_r) = 0$, with (36), one has

$$(\tau_i^{(d)} \otimes I_r)' \Gamma_{ci}(\tau_i^{(d)} \otimes I_r) < 0,$$

it implies that

$$\begin{aligned}
W_i(t)A'_i + A_iW_i(t) + B_iQ_i(t) + Q'_i(t)B'_i - \mathcal{D}^+W_i(t) \\
+ \lambda_i^\diamond W_i(t) < 0. \tag{41}
\end{aligned}$$

Multiply both sides of (41) with $Z_i(t) = W_i^{-1}(t)$, and substitute $Q_i(t) = K_i(t)W_i(t)$ in (41), then one has

$$\begin{aligned}
A'_{ci}W_i^{-1}(t) + W_i^{-1}(t)A_{ci} - W_i^{-1}(t)\mathcal{D}^+W_i(t)W_i^{-1}(t) \\
+ \lambda_i^\diamond W_i^{-1}(t) < 0. \tag{42}
\end{aligned}$$

Since $\mathcal{D}^+W^{-1}(t) = -W^{-1}(t)\mathcal{D}^+W(t)W^{-1}(t)$, then (42) can be rewritten as

$$A'_{ci}Z_i(t) + Z_i(t)A_{ci}(t) + \mathcal{D}^+Z_i(t) + \lambda_i^\diamond Z_i(t) < 0. \tag{43}$$

Then, with (41) and (43), according to Theorem 2, the λ^* -exponential stability of the closed-loop system is established. \square

Remark 3. Notice that the controller gains obtained with Theorem 3 are time-varying, which is different from the one in Li, Lam, and Chung (2015). In that work, constant controller gains were used for different subsystems by employing linear time-varying Lyapunov function. The condition in Li, Lam, and Chung (2015) was non-convex, which could not be solved directly. An iterative algorithm is proposed to obtain the controllers gain, which is complicated and dependent on initial conditions. In Theorem 3, the condition is less conservative and the controller can be solved directly, which is more applicable to general situations. \square

Remark 4. It is worth mentioning that controller designed in Theorem 3, similar to those proposed in Li, Lam, and Chung (2015), can be used to stabilize the periodic piecewise linear system with unstabilizable subsystems. In Li, Lam, and Chung (2015), controllers were only designed for stabilizable subsystems, and no controllers were designed for unstabilizable subsystem in order to avoid resulting in large controller gain during the iteration. However, in this method, controllers are designed for all subsystems. For the unstabilizable subsystem, the controllable part can be stabilized under the designed controller. \square

In order to facilitate to verify the merit of this process, two approaches are proposed based on Theorem 3. **Approach I:** no controller is designed for unstabilizable subsystem, as in Li, Lam, and Chung (2015). **Approach II:** all subsystem as designed with controllers. It can be noticed that Approach I is a special case of Algorithm II.

5. Numerical examples

In this section, two numerical examples are used to verify the effectiveness of the proposed design approach. Example 1 is used to verify the merits of our method when compared with that in Li, Lam, and Chung (2015), Example 2 is used to verify the merits designing controllers for unstabilizable subsystems in Theorem 3.

Example 1. Consider a periodic piecewise system consisting three subsystems with $T_p = 2$ s and $t_1 = 0.5$ s, $t_2 = 1.3$ s, $t_3 = 2$ s, the subsystems are described by

$$\begin{aligned}
A_1 & = \begin{bmatrix} -2.1 & 0.6 \\ 0 & 0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.8 & 0.1 \\ 0.2 & 0.6 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 2.5 & 1.8 \\ 1.6 & -3.5 \end{bmatrix}, \\
B_1 & = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.1 \\ 3 \end{bmatrix}
\end{aligned}$$

and the system initial state is $x(0) = [0.01, 0.2]'$. Obviously, all subsystems are non-Hurwitz stable and the first system is unstabilizable. Using Theorem 1 in Li, Lam, and Chung (2015), one can see that this periodic piecewise system is unstable. In the following, we will design stabilizing controllers for this system with Approach I, choosing $d = 1$, construct a periodic continuous matrix polynomial $W(t)$ for $t \in [0, T_p)$ with degree 2. Since the first subsystem is unstabilizable, no controller is designed for this system. Choose $\lambda_1^\diamond = -2$, $\lambda_2^\diamond = 0.5$, $\lambda_3^\diamond = 2$, then λ^* can be chosen as 0.11, according to Approach I, the controller gain is obtained and shown in Fig. 1. The free variables are obtained as $\varepsilon_{c1} = -0.4908$, $\varepsilon_{c2} = 1.7860$, $\varepsilon_{c3} = 5.2048$, $\zeta_{c1} = -3.1331$, $\zeta_{c2} = 2.0889$, $\zeta_{c3} = -0.1258$.

Then with the obtained controller, the system state components are shown in Fig. 2, one can observe that the system is stabilized under the obtained controller.

For the approach proposed in Li, Lam, and Chung (2015), a piecewise linear Lyapunov matrix is given in interpolation formulation, constant controller gain K_i is allocated for different subsystems. Choosing initial condition $\lambda_1^{\diamond 1} = -100$, $\lambda_2^{\diamond 1} = 0$, $\lambda_3^{\diamond 1} = 0$, $K_1^1 = [0 \ 0]$, $K_2^1 = [-1 \ -1]$, $K_3^1 = [-3 \ -0.5]$, the conditions

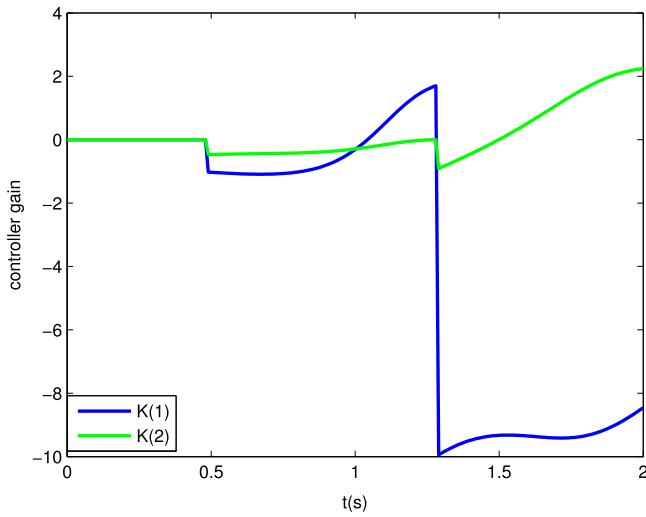


Fig. 1. Variation of $K_2(t), K_3(t)$ over a period with Approach I (Example 1).

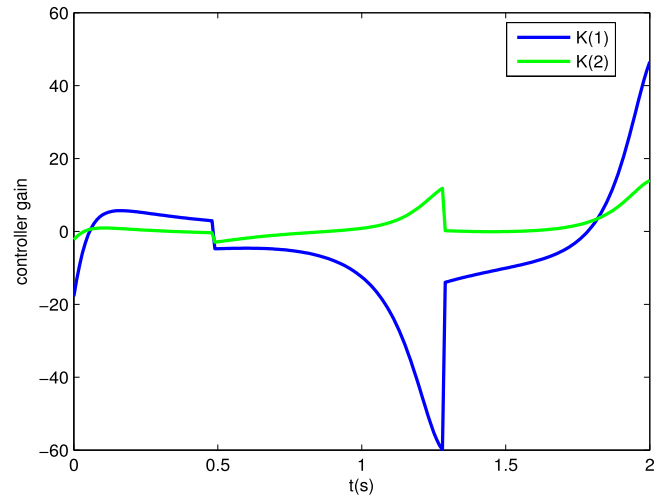


Fig. 3. Variation of $K(t)$ over a period with Approach II (Example 2).

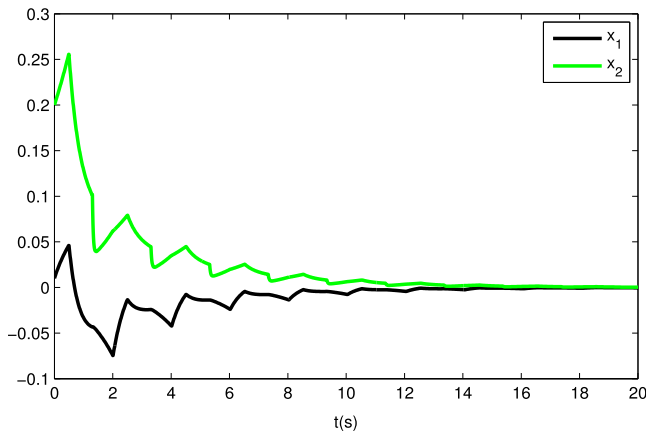


Fig. 2. Trajectory of system state under stabilizing controller with Approach I (Example 1).

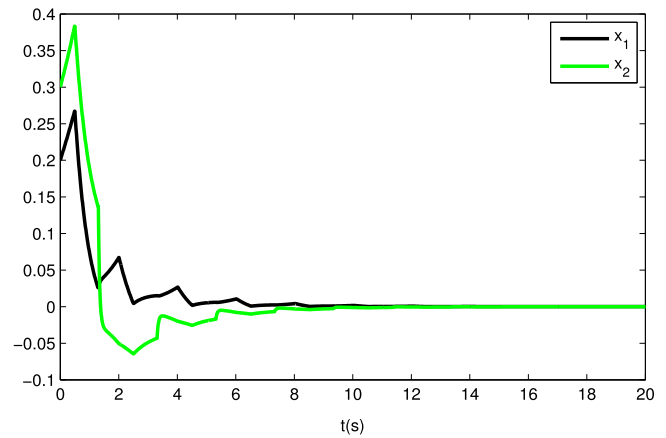


Fig. 4. Trajectory of system state under stabilizing controller with Approach II (Example 2).

are infeasible. It can be observed that this approach is dependent on the initial condition chosen. Moreover, the linear interpolation formulation of the Lyapunov matrix is more restrictive on the controller structure. As mentioned in Remarks 2 and 3, noting that the proposed method has larger admissible ranges of λ_i^\diamond and allows time-varying controller gains. The method proposed in this paper is thus more effective.

Example 2. Consider a periodic piecewise linear system consisting three subsystems described as follows:

$$A_1 = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0.4 \\ 0 & 0.4 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 3 & 1.5 \\ 1.6 & 3 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

with $T_p = 2$ s and $T_1 = 0.5$ s, $T_2 = 0.8$ s, $T_3 = 0.7$ s, $x(0) = [0.1, 0.2]'$. It can be observed that all subsystems are non-Hurwitz and the second subsystem is unstabilizable. It is easy to conclude this periodic system is unstable (Li, Lam, & Chung, 2015). In the following, we will design controllers according to Theorem 3 with both algorithms. Choosing $\lambda_1^\diamond = -1, \lambda_1^\diamond = -2, \lambda_1^\diamond = 4$, then λ^* can be chosen as 0.175. According to Approach I, no controller is designed for the second subsystem, the condition is infeasible, controllers cannot be obtained to stabilize the system. According to

Approach II, controller is also designed for the second subsystem. The variation of the controller gains are shown in Fig. 3 based on the obtained $W^H(t), Q_i^H(t)$, the system state components of the closed-loop system are shown in Fig. 4.

The free variables are obtained as $\varepsilon_{c1} = 6.5050, \varepsilon_{c2} = 21.5157, \varepsilon_{c3} = 2.5960, \zeta_{c1} = -2.9967, \zeta_{c2} = 45.0456, \zeta_{c3} = -13.8201$. It can be seen that the closed-loop system is stable under this set of controllers. One may conclude that it is more effective to stabilize a system with Approach II when compared with Approach I. It should also be noticed that in this example, a negative λ_1^\diamond is associated with the first subsystem which is stabilizable. It indicates that in our method, we provide more relaxed conditions than those in Li, Lam, and Chung (2015).

6. Conclusion

In this paper, new conditions of exponential stability and stabilization problems for periodic piecewise linear systems subsystems are proposed. Periodic continuous Lyapunov function is constructed with time-dependent Lyapunov matrix polynomial. Based on the square matricial representation and sum of squares form of the homogeneous Lyapunov matrix polynomial, exponential stability condition is obtained first. Stabilizing controllers are then designed with time-varying polynomial controller gains,

which is more applicable in practice. Numerical examples are included to illustrate the effectiveness of the proposed method.

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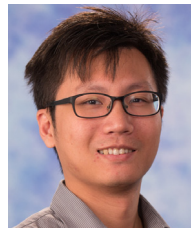


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